Minicourse: A strong version of the Sims conjecture on finite primitive permutation groups

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In the mid 1960s, Ch. Sims stated the following conjecture: There exists a function $\varphi : \mathbb{N} \longrightarrow \mathbb{N}$ such that, if G is a primitive permutation group on a finite set X, G_x is the stabilizer in G of a point x from X, and d is the length of any G_x-orbit on $X \setminus \{x\}$, then $|G_x| \leq \varphi(d)$.

Some progress toward to prove this conjecture had been obtained in papers of Sims (1967)), Thompson (1970), Wielandt (1971), Knapp (1973, 1981)), Fomin (1980). But only with the use of the classification of finite simple groups, the validity of the conjecture was proved by Cameron, Praeger, Saxl and Seitz (1983).

The Sims conjecture can be formulated using graphs as follows. For an undirected connected graph Γ (without loops or multiple edges) with vertex set $V(\Gamma)$, $G \leq Aut(\Gamma)$, $x \in V(\Gamma)$, and $i \in \mathbb{N} \cup \{0\}$, denote by $G_x^{[i]}$ the elementwise stabilizer in G of the (closed) ball of radius i of the graph Γ centered at x in the natural metric d_Γ on $V(\Gamma)$. Then the Sims conjecture is equivalent to the following statement: There exists a function $\psi : \mathbb{N} \cup \{0\} \longrightarrow \mathbb{N}$ such that, if Γ is an undirected connected finite graph and G is its automorphism group acting primitively on $V(\Gamma)$, then $G_x^{[\psi(d)]} = 1$ for $x \in V(\Gamma)$, where d is the valency of the graph Γ.

The lecturer jointly with Trofimov in 1999 obtained the following strengthened version of the Sims conjecture: If Γ is an undirected connected finite graph and G its automorphism group acting primitively on $V(\Gamma)$, then $G_x^{[6]} = 1$ for $x \in V(\Gamma)$.

New we investigate the more general problem of describing all pairs (Γ, G), where Γ is an undirected connected finite graph, G is its automorphism group acting primitively on $V(\Gamma)$ and $G_x^{[2]} \neq 1$ for $x \in V(\Gamma)$.

The aim of the lectures is to discuss obtained results and some methods of their proofs.

It contains 2 lectures.