Krasovskii Institute of Mathematics and Mechanics UB RAS
Ural Federal University
named after the first President of Russia B.N. Yeltsin

Groups and Graphs, Metrics and Manifolds

Yekaterinburg, Russia, 22–30 July 2017

Abstracts

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Contents

General Information

Krasovskii Institute of Mathematics and Mechanics of Ural Branch of Russian Academy of Sciences and Ural Federal University named after the first President of Russia B.N. Yeltsin organize the International Conference and PhD-Master Summer School "Groups and Graphs, Metrics and Manifolds" (G2M2).

All scientific activities take place at Ural Federal University, 4 Turgeneva St., Yekaterinburg, Russia on July 22–30, 2017.

G2M2 aims to cover modern aspects of group theory, graph theory, 3-manifold theory (including knot theory), and some aspects of optimization theory.

The official language of the event is English.

Scientific committee:
Alexander Makhnev (*chair IMM*), Sergey Matveev (*co-chair CSU*), Alexander Gavrilyuk, Sergey Goryainov, Vladislav Kabanov, Anatoly Kondrat'ev, Denis Krotov, Natalia Maslova, Alexander Mednykh, Alexander Osipov, Ludmila Tsiovkina, Mikhail Volkov.

Organizing committee:
Vladislav Kabanov (*chair IMM*), Mikhail Volkov (*co-chair UrFU*), Vitaly Baransky, Ivan Belousov, Sergey Goryainov, Anatoly Kondrat'ev, Elena Konstantinova, Anton Konygin, Alexander Makhnev, Natalia Maslova, Tat'yana Senchonok, Marianna Zinov'eva.

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Sergey Goryainov, Elena Konstantinova, Klavdija Kutnar, Alexander Makhnev, Natalia Maslova, Alexander Mednykh.

Partners:
Sobolev Institute of Mathematics of Siberian Branch of Russian Academy of Sciences
Novosibirsk State University

Website:
g2m2.imm.uran.ru

Conference Program

Saturday, July 22

10:00 - 18:00	Registration: *Ural Federal University, 51 Lenina Avenue, Yekaterinburg*

Sunday, July 23

10:00 - 10:50	Mikhail Volkov: *Algebraic properties of monoids of diagrams and 2-cobordisms*
10:50 - 11:00	*Break*
11:00 - 11:50	Tatsuro Ito: *Terwilliger algebras of (P and Q)-polynomial schemes I*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Jack Koolen: *Trees, Lattices and Hoffman graphs I*
13:00 - 13:15	*Conference Photo*
13:15 - 14:30	*Lunch*
14:30 - 15:20	Sergey Shpectorov: *Non-existence of some strongly regular graphs via the unit vector representation*
15:20 - 15:30	*Break*
15:30 - 16:20	Michael Khachay: *Deterministic and Randomized Approximation Algorithms for the Traveling Salesman Problem and Its Generalizations*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
18:10 - 19:00	*Discussion*

Monday, July 24

10:00 - 10:50	Akihiro Munemasa : *A matrix approach to Yang multiplication I*
10:50 - 11:00	*Break*
11:00 - 11:50	Jack Koolen: *Trees, Lattices and Hoffman graphs II*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Alexandre Zalesski: *Minicourse I, Lecture 1*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Alexander Ivanov: *Graphs, Geometries, and Amalgams*
15:20 - 15:30	*Break*
15:30 - 16:30	*Contributed Talks*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
16:50 - 18:10	*Problem solving*

Tuesday, July 25

10:00 - 10:50	Akihiro Munemasa : *A matrix approach to Yang multiplication II*
10:50 - 11:00	*Break*
11:00 - 11:50	Danila Revin: *Pronormality of subgroups in finite groups*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Alexandre Zalesski: *Minicourse I, Lecture 2*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Andrey Vasil'ev: *On the k-closure of a permutation group*
15:20 - 15:30	*Break*
15:30 - 16:30	*Contributed Talks*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
18:10 - 21:00	*Conference Dinner*

Wednesday, July 26

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10:50 - 11:00	*Break*
11:00 - 11:50	Anatoly Kondrat'ev: *Minicourse II, Lecture 1*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Alexandre Zalesski: *Minicourse I, Lecture 3*
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14:30 - 15:20	*Open Problems Session*
15:20 - 15:30	*Break*
15:30 - 16:30	*Contributed Talks*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
18:10 - 19:00	*Problem solving*

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10:50 - 11:00	*Break*
11:00 - 11:50	Anatoly Kondrat'ev: *Minicourse II, Lecture 2*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Alexandre Zalesski: *Minicourse I, Lecture 4*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Tatsuro Ito: *Terwilliger algebras of (P and Q)-polynomial schemes II*
15:20 - 15:30	*Break*
15:30 - 16:30	*Contributed Talks*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
18:10 - 19:00	*Problem solving*

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10:50 - 11:00	*Break*
11:00 - 11:50	Alexander Mednykh: *Jacobians of circulant graphs and their generalisations*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Vladimir Trofimov: *Symmetrical extensions of graphs I*
12:30 - 14:30	*Lunch*
14:30 - 15:20	Vladimir Trofimov: *Symmetrical extensions of graphs II*
15:20 - 15:30	*Break*
15:30 - 16:30	*Contributed Talks*
16:30 - 16:50	*Coffee break*
16:50 - 18:10	*Contributed Talks*
18:10 - 19:00	*Problem solving*

Saturday, July 29

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11:00 - 11:50	Yaokun Wu: *Lipschitz polytopes of metric spaces*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Shaofei Du: *Recent Developments in Regular Maps*
13:00 - 13:10	*Break*
13:10 - 14:00	Alexander Makhnev: *Eigenvalues of distance-regular graphs*
14:00	*Closing*

Sunday, July 30

Leaving the Conference

G2 Conferences

G2-events are International Conferences and PhD–Master Summer Schools on Graphs, Groups, and related topics. The main goal of the events is to bring together students, young researchers, scientists, and experts to exchange knowledge and results in a broad range of topics relevant to graph theory and group theory with connections to combinatorics, topology, geometry, coding theory, automata and formal language theory, algorithm theory, network analysis, and applications.

The first event of the series, "Graphs and Groups, Cycles and Coverings" (G2C2), was held on September 23−26, 2014, in Akademgorodok, Novisibirsk, Russia, in the frame of the International cooperation between Slovenia and Russia in 2014 − 2015 with the support of the Slovenian Research Agency. Five slovenian mathematicians from University of Primorska, Koper, presented excellent talks on group actions on combinatorial objects. The talks were given by Klavdija Kutnar, Tomaž Pisanski, Aleksander Malnič, István Kovács, and Istvăn Estělyi. It was a small workshop with no more than 30 participants from Novosibirsk, Moscow, Yekaterinurg, Chelyabinsk, however, it was successful enough. It was organized by Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences and Novosibirsk State University. The main organizers of the workshop were Elena Konstantinova, Klavdija Kutnar, and Alexander Mednykh.

After a success of G2C2, the series of G2–events was conceived by the steering committee which includes Sergey Goryainov, Elena Konstantinova, Klavdija Kutnar, Alexander Makhnev, Natalia Maslova, and Alexander Mednykh. In the winter of 2015 it was decided to have the next event in Yekaterinburg.

The second event, The International Conference and PhD Summer School "Groups and Graphs, Algorithms and Automata" (G2A2), was organized by Krasovskii Institute of Mathematics and Mechanics of Ural Branch of Russian Academy of Sciences and Ural Federal University named after the first President of Russia B.N. Yeltsin with Alexander Makhnev as the chair of the scientific committee. The main organizers of G2A2 were Vladislav Kabanov, Mikhail Volkov, Natalia Maslova and Sergey Goryainov. It was held on August 9 − 15, 2015, in the recreation area Ivolga which is located near Yekaterinburg, Russia. The topic of the event included all the branches of group theory, graph theory, automata and formal language theory, and algorithm theory. The scientific program of G2A2 event consisted of minicourses, plenary and contributed talks. Klavdija Kutnar and Dragan Marušič presented the exciting course on Graphs and their Automorphism Groups. Tomaž Pisanski teached students on Symmetries in Graphs with Python and Sage. After taking that course many students started to use Python as principal software to write papers, reports, etc. With a great enthusiasm, Mikhail Volkov explained a problem on Synchronizing finite automata which everyone can understand but nobody can solve (so far). The team of main speakers was presented by Bernhard Amberg (Johannes Gutenberg University Mainz), Tatsuro Ito (Anhui University), Lev Kazarin (Demidov Yaroslavl State University), Jack Koolen (University of Science and Technology of China), Vladimir Levchuk (Siberian Federal University), Nadezhda Timofeeva (Demidov Yaroslavl State University), Evgeny Vdovin (Sobolev Institute of Mathematics). Around 100 experts on finite group theory, graph theory, algebraic combinatorics, automata and formal language theory, and algorithm theory from 7 countries (Belarus, China, Germany, Hungary, Slovenia, Russia, and Ukraine) participated in the G2A2–event.

The third event, The International Conference and PhD–Master Summer School on "Graphs and Groups, Spectra and Symmetries" (G2S2), was held on August 15 − 28, 2016, in Novosibirsk, Russia. The G2S2–event was organized by Sobolev Institute of Mathematics and Novosibirsk State University with cooperation of the Krasovskii Institute of Mathematics and Mechanics, and with Elena Konstantinova, Denis Krotov, and Alexander Mednykh as main organizers. The G2S2–event was supported by the Russian Foundation for Basic Research, grant 16−31−10290, and Novosibirsk State University, Project 5−100. More than 110 experts on finite group theory, graph theory, algebraic combinatorics, low–dimensional geometry and topology from 19 countries (Brazil, Canada, China, Czech Republic, Finland, Germany, Hungary, India, Iran, Israel, Italy, Japan, Slovakia, Slovenia, South Korea, Taiwan, United Kingdom, USA and Russia) participated in the G2S2–events. Young scientists, PhD students, graduate and undergraduate students were presented by 75 participants. The scientific program of G2S2–events consisted of minicourses, plenary and contributed talks, open problems session. Four minicourses, each containing eight 50–minutes lectures, were given by Lih-Hsing Hsu (Providence University, Taichung, Taiwan), Bojan Mohar (University of Ljubljana, Slovenia; Simon Fraser University, Vancouver, Canada), Alexander A. Ivanov (Imperial College London, UK) and Ted Dobson (Mississippi State University, USA; University of Primorska, Slovenia). Twenty main speakers gave brilliant talks on algebraic combinatorics, on isomorphism problem for graphs, Cayley graphs and Cayley combinatorial objects, on colour–preserving automorphisms of Cayley graphs, on integral graphs and Cayley graphs, on characterization of the Grassmann graphs and on applications of Hoffman graphs, on symmetry properties of combinatorial objects, on the classification of association schemes, on codes obtained from some graphs and finite geometries, on partial geometry with given parameters, on topological graph theory problems, on the lit-only sigma game and some mathematics around, on plateaued Boolean functions with the same spectrum support. As it was noticed by Alexander A. Ivanov, the source of many ideas discussed during the conference was Com^{2}Mac, The Combinatorial and Computational Mathematics Center, POSTECH, Korea (1999 – 2008), supervising by Professor Jin Ho Kwak (Beijing Jiaotong University, China), who participated at the G2S2–events. 15 participants of G2S2 were the members of this institution for a while. The list of the main speakers is given by Anton Betten, Shaofei Du, Alexander Gavrilyuk, Mitsugu Hirasaka, Tatsuro Ito, Lev Kazarin, Jack Koolen, Klavdija Kutnar, Jin Ho Kwak, Dragan Marušič, Akihiro Munemasa, Mikhail Muzychuk, Roman Nedela, Patric Patric Östergård, Ilia Ponomarenko, Yuriy Tarannikov, Andrey Vasil'ev, Yaokun Wu, Matan Ziv-Av. The participants of the event were invited to submit a research paper based on the talks for the proceedings to appear as a Special Issue of the Siberian Electronic Mathematical Reports (SEMR). Totally, 12 papers were published in SEMR in 2016 (see http://math.nsc.ru/conference/g2/g2s2/papers.html). To find more details on G2S2–event, see [1]. Video lectures of all courses and plenary talks are published online by Youtube, see http://math.nsc.ru/conference/g2/g2s2/video.html.

The current event, The International Conference and PhD-Master Summer School on "Groups and Graphs, Metrics and Manifolds" (G2M2), is organized by Krasovskii Institute of Mathematics and Mechanics of Ural Branch of Russian Academy of Sciences and Ural Federal University named after the first President of Russia B.N. Yeltsin. The main goal of this event is to bring together young researchers and experts in the field of group theory, graph theory, and 3-manifold theory, including knot theory. The scientific program consists of minicourses, plenary and contributed talks.

We are looking forward to have you as one of the participants of G2M2–events.

Enjoy the art of mathematics with us!

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Minicourses

On structure of finitary permutation groups

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The aim of the lectures is to present three distinct approaches to investigation of a structure of finitary permutation groups. The first approach is based on the standard permutation notions such as Orbits, Stabilizers, Blocks, and so on. The second approach is topological. The third approach is geometric, it gives us a way to construct finitary permutation groups as finitary automorphism groups of some graphs.

The minicourse contains 2 lectures.

A strong version of the Sims conjecture on finite primitive permutation groups

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In the mid 1960s, Ch. Sims stated the following conjecture: There exists a function $\varphi : \mathbb{N} \longrightarrow \mathbb{N}$ such that, if G is a primitive permutation group on a finite set X, G_x is the stabilizer in G of a point x from X, and d is the length of any G_x-orbit on $X \setminus \{x\}$, then $|G_x| \leq \varphi(d)$.

Some progress toward to prove this conjecture had been obtained in papers of Sims (1967), Thompson (1970), Wielandt (1971), Knapp (1973, 1981), Fomin (1980). But only with the use of the classification of finite simple groups, the validity of the conjecture was proved by Cameron, Praeger, Saxl and Seitz (1983).

The Sims conjecture can be formulated using graphs as follows. For an undirected connected graph Γ (without loops or multiple edges) with vertex set $V(\Gamma)$, $G \leq Aut(\Gamma)$, $x \in V(\Gamma)$, and $i \in \mathbb{N} \cup \{0\}$, denote by $G_x^{[i]}$ the elementwise stabilizer in G of the (closed) ball of radius i of the graph Γ centered at x in the natural metric d_Γ on $V(\Gamma)$. Then the Sims conjecture is equivalent to the following statement: There exists a function $\psi : \mathbb{N} \cup \{0\} \longrightarrow \mathbb{N}$ such that, if Γ is an undirected connected finite graph and G is its automorphism group acting primitively on $V(\Gamma)$, then $G_x^{[\psi(d)]} = 1$ for $x \in V(\Gamma)$, where d is the valency of the graph Γ.

In 1999, the lecturer jointly with Trofimov obtained the following strengthened version of the Sims conjecture: If Γ is an undirected connected finite graph and G its automorphism group acting primitively on $V(\Gamma)$, then $G_x^{[6]} = 1$ for $x \in V(\Gamma)$.

Now we investigate the more general problem of describing all pairs (Γ, G), where Γ is an undirected connected finite graph, G is its automorphism group acting primitively on $V(\Gamma)$ and $G_x^{[2]} \neq 1$ for $x \in V(\Gamma)$.
The aim of the lectures is to discuss obtained results and some methods of their proofs.
The minicourse contains 2 lectures.

Introduction to the character theory of finite groups of Lie type

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Character theory of finite groups of Lie type is an advanced area of representation theory of finite groups containing numerous fascinating results. Significance of the theory for group theory derives from the fact that the majority of simple groups are groups of Lie type, so one cannot ignore these groups when approaching problems of general nature.

The aim of the lectures is to focus on the key elements of the theory in order to help beginners to orient in the area and understand the central ideas of the theory.

Some knowledge of classical theory of finite group representation is required, including notions of irreducible and induced representations, as well as basic standard results of character theory (inner product of characters, orthogonality relations etc).

The minicourse contains 4 lectures.

Plenary Talks

Recent Developments in Regular Maps

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A 2-cell embedding of a graph into an orientable or nonorientable closed surface is called regular if its automorphism group acts regularly on its arcs and flags respectively. One of central problems in topological graph theory is to classify regular maps by given underlying graphs or automorphism groups. In this talk, we shall present some recent results in regular maps.

Terwilliger algebras of (P and Q)-polynomial schemes

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It was nearly forty years ago that Eiichi Bannai proposed the classification of (P and Q)-polynomial schemes. He interpreted P-polynomial schemes as a combinatorial analogue of compact 2-point homogeneous spaces, Q-polynomial schemes as that of compact symmetric spaces of rank 1. Note that compact symmetric spaces of rank 1 are classified by Élie Cartan, and that by Hsien Chung Wang's theorem, compact 2-point homogeneous spaces are compact symmetric spaces of rank 1 and vice versa. Bannai made a list of (P and Q)-polynomial schemes and conjectured that (1) if a (P and Q)-polynomial scheme has diameter large enough, it must be in the list or have a 'relative ' in the list, (2) P-polynomial schemes are Q-polynomial schemes and vice versa, if they are primitive and have diameter large enough. Since then, the classification of (P and Q)-polynomial schemes with diameter sufficiently large has been one of the central problems in algebraic combinatorics.

The Terwilliger algebra was introduced by Paul Terwilliger early in the 1990s for a commutative association scheme; it was originally called a subconstituent algebra by himself. It can be thought of a combinatorial analogue of the centralizer algebra for the one-point stabilizer of the automorphism group, whereas the Bose-Mesner algebra is the one for the automorphism group itself. It is generated by the Bose-Mesner algebra and the dual of it. Hence it is non-commutative and much larger than the Bose-Mesner algebra. Terwilliger developed a deep theory of representations of the Terwilliger algebras for (P and Q)-polynomial schemes. The theory is not only a key tool of the classification of (P and Q)-polynomial schemes but also it has interesting interactions with many other branches of mathematics such as Lie theory, quantum groups, statistical mechanics.

In my talks, I will first explain the representation theory of Terwilliger algebras and then discuss the present status of the classification of (P and Q)-polynomial schemes.

Graphs, Geometries, Amalgams

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Let Γ be a bipartite graph with edge-transitive automorphism group G. The vertices in one part are called *lines* and they are of valency 3, the vertices in the other part are called *points* and they are of valency $2^n - 1$ for $n \geq 2$. The stabilizer of a point induces on its neighbors the natural 2-transitive action of $L_n(2)$. The ultimate goal is to classify the amalgams $\{G(p), G(l)\}$ formed by the stabilizers of adjacent point-line pairs (p, l). For $n = 2$ there 15 such amalgam as proved in the ground breaking paper by D.Goldschmidt [2]. For every Goldschmitd's amalgam the order of $G(p)$ divides $2^7 \cdot 3$ and the largest amalgam is realized in the automorphism group of the Mathieu group M_{12}. For $n \geq 3$ the known examples come from point-line incident graphs of classical and sporadic flag-transitive geometries. The classical examples are associated with the linear and symplectic geometries over $GF(2)$ along with the Hamming graphs over the alphabet with 3 letters and automorphism group $S_3 \wr L_n(2)$. The sporadic examples include the exceptional A_7-geometry, Cooperstein's geometry of $G_2(3)$, and tilde geometries of the Mathieu group M_{24}, the Held group He, the Conway group Co_1, culminating at the Monster group M, where

$$G(p) \cong 2^6.2^5.2^{10}.2^{10}.2^5.L_5(2).$$

There is also an important example associated with the locally truncated C_4-geometry of M_{24}. The similar problem with lines having valency 2 was accomplished in [2] where V.I. Trofimov's results announced in [3] and published in numerous subsequent articles had played an essential role along with the classification of Petersen geometries completed by the authors of [2]. A special case of $n = 3$ problem with lines of valency 3 was settled in [4], where the amalgams coming from M_{24}- and He-geometries were characterized (and proved to be isomorphic to each other), and a new amalgam realized in A_{16} was discovered. Our general strategy is to recover a geometry from the graph and to apply the amalgam method to tackle down the possible residues. The classification of tilde geometries by the authors of [2] is expected to be the final accord.

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Deterministic and Randomized Approximation Algorithms for the Traveling Salesman Problem and Its Generalizations

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The Traveling Salesman Problem (TSP) is the classic combinatorial optimization problem introduced by G.Dantzig and J.Ramser in their seminal paper [1], where the authors consider a scheduling problem of gasoline trucks servicing a network of gas-stations. It is curious, but the authors believed that the problem they stated is tractable and can be easily solved to optimality for any instance length. Now, thanks to R.Karp and C.Papadimitriu, we know that TSP is strongly NP-hard [2] and remains intractable even in the Euclidean plane [3]. Therefore, due to the well-known $P \neq NP$ conjecture, efficient optimal algorithms for TSP will hardly be designed ever. Furthermore, as shown in [4], in its general setting, TCP can not be approximated efficiently with any reasonable accuracy, since it has no $O(2^n)$-ratio polynomial time approximation algorithms unless $P = NP$. Meanwhile, in more specific (but acceptable for numerous applications) settings, there are known many promising approximation results, among them are famous 3/2-approximation algorithm by N.Christofides [5] for the metric TSP and S.Arora's Polynomial Time Approximation Schemes (PTAS) for fixed dimensional Euclidean spaces [6].

Recently it was proven [7] that some known and valuable for applications generalizations of TSP have the similar complexity status and approximability behavior. In this talk we consider the k-Size Cycle Cover Problem, where a given edge-weighted complete (di)graph should be covered by k vertex-disjoint cycles of minimum total weight. For $k = 1$, this problem coincides with TSP and intractable. On the other hand, for unbound k, the problem is equivalent to the minimum weight perfect matching problem and can be solved to optimal in polynomial time. We show that, for any fixed k, k-SCCP is strongly NP-hard even in the plane, inapproximable in general setting, belongs to APX for any metric and has EPTAS in d-dimensional Euclidean space for any fixed $d > 1$ [8].

Another topic of this talk deals with randomized and asymptotically optimal algorithms for max-TSP and SCCP proposed recently by E. Gimadi et al. (see, e.g. [9]).

This survey is supported by RSF, grant no. 14-11-00109.

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Trees, Lattices and Hoffman graphs

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Let G be a connected graph. With an eigenvalue of G we will mean an eigenvalue of its adjacency matrix. As the adjacency matrix is symmetric and real, all its eigenvalues are real. We will mainly look at the minimal eigenvalue of G.

We say that a graph G is a *generalized line graph* if its adjacency matrix A satisfies the following equation $A + 2I = N^T N$ where each entry of N is integral. (Note that it follows that N has only entries $0, \pm 1$.)
In 1976, Cameron, Goethals, Seidel and Shult showed that a connected graph with smallest eigenvalue at least -2 either is a generalized line graph or the number of vertices is at most 36.

In these talks, I will discuss connected graphs with smallest eigenvalue at least -3.

Let G be a connected graph with smallest eigenvalue at least -3 and A its adjacency matrix. Then $A+3I = N^T N$ for some real matrix N. We will discuss properties of the lattice generated by the columns of N.

Eigenvalues of distance-regular graphs

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This talk is based on joint works with Ivan Belousov and Marina Nirova.

Let Γ be a distance-regular graph of diameter $d = 2e + 1$, e be a positive integer, and C be an e-code in Γ. Then $|C| \leq p^d_{dd} + 2$. If the equality is achieved then C is called a maximal e-code.

Let C be a maximal e-code in Γ. Then $c_d \geq a_d p^d_{dd}$. If the equality is achieved then C is called a locally regular e-code.

Let C be an e-code in Γ. Then $|C| \leq k_d/\sum_{i=0}^{e} p^d_{id} + 1$. If the equality is achieved then C is called a last subconstituent perfect e-code.

In [1, Proposition 5] the following proposition was proved.

Proposition 1. *Let Γ be a distance-regular graph of diameter 3 containing a maximal 1-code C that is both locally regular and last subconstituent perfect. Then the intersection array of Γ is either $\{a(p+1), cp, a+1; 1, c, ap\}$ or $\{a(p+1), (a+1)p, c; 1, c, ap\}$, where $a = a_3$, $c = c_2$, and $p = p^3_{33}$.*

In the last case Γ is a Shilla graph with $b_2 = c_2$. In this case $a = a_3$ divides k and for $b = b(\Gamma) = k/a$ we have $p^3_{33} = b - 1$. A Shilla graph with $b_2 = c_2$ is Q-polynomial if and only if $\theta_3 = -b(b + 1)/2$. In this case $b = 2r$, $c_2 = (t + r)r$, and the intersection array of Γ is $\{2rt(2r + 1), (2r - 1)(2rt + t + 1), r(r + t); 1, r(r + t), t(4r^2 - 1)\}$. Moreover, in view of [2, Proposition 4.6.3] for any vertex $u \in \Gamma$ the subgraph $\Gamma_3(u)$ is an antipodal distance-regular graph with the intersection array $\{t(2r+1), (2r-1)(t+1), 1; 1, t+1, t(2r+1)\}$ and $t \leq 2r(r+1)(2r-1)-r$. Note that in the case $t = r$ we have $r = 1$.

In this talk we discuss the following results on Shilla graphs with $b_2 = c_2$:

(1) a Shilla graph with $b_2 = c_2$ and with a noninteger eigenvalue has the intersection array $\{b^2(b-1)/2, (b-1)(b^2 - b + 2)/2, b(b - 1)/4; 1, b(b - 1)/4, b(b - 1)^2/2\}$;

(2) Shilla graphs with $b_2 = c_2$ and $b \in \{4, 5\}$ were classified;

(3) exact formulas for multiplicities eigenvalues of Shilla graphs with $b_2 = c_2$ were founded;

(4) a new infinite series of feasible intersection arrays of hypothetic Q-polynomial Shilla graphs with $b_2 = c_2$ was founded: $\{2r(2r^2 - 1)(2r + 1), (2r - 1)(2r(2r^2 - 1) + 2r^2), r(2r^2 + r - 1); 1, r(2r^2 + r - 1), (2r^2 - 1)(4r^2 - 1)\}$;

(5) graphs with following intersection arrays do not exist:

(*i*) $\{2(3s - 1)^3(6s - 1), 9s(2s - 1)(18s^2 - 15s + 1), 3s(3s - 1)^2; 1, 3s(3s - 1)^2, 3(3s - 1)^2(2s - 1)(6s - 1)\}$, where $s \notin \{1, 2, 4\}$;

(*ii*) $\{6(3s+1)^3(2s+1), (3s+2)(6s+1)(18s^2+9s+2), (3s+2)(3s+1)^2; 1, (3s+2)(3s+1)^2, 3(3s+1)^2(2s+1)(6s+1)\}$, where $s > 0$;

(*iii*) $\{20t, 3(5t + 1), 2(t + 2); 1, 2(t + 2), 15t\}$, where $t = 16, 22, 70$;

(*iv*) $\{42t, 5(7t + 1), 3(t + 3); 1, 3(t + 3), 35t\}$, where $t = 12, 27, 57, 117$;

(*v*) $\{4r^2(r-1)(2r+1), (2r-1)(4r^2(r-1) + 2r(r-1) + 1), r^2(2r-1); 1, r^2(2r-1), 2r(r-1)(4r^2-1)\}$, where $r \neq 2$.

This part of our work was supported by the grant of Russian Science Foundation, project no. 14-11-00061-P.

Let Γ be a distance-regular graph of diameter 3 with $\theta_2 = -1$. If the intersection array of Γ is $\{a(p + 1), cp, a + 1; 1, c, ap\}$ then $\theta_2 = -1$ and Γ_3 is a pseudogeometric graph for $GQ(p + 1, a)$. Note that in the case $a = c + 1$ and the graph $\bar{\Gamma}_2$ is a pseudogeometric graph for $pG_2(p + 1, 2a)$.

Proposition 2. *Let Γ be a distance-regular graph of diameter 3. Then Γ_3 is a strongly regular graph if and only if $\theta_2 = -1$. In this case $\bar{\Gamma}_3$ is a pseudogeometric graph for $pG_{c_3}(k, b_1/c_2)$.*

Theorem 1. *Let Γ be a primitive distance-regular graph of diameter 3, where Γ_2 and Γ_3 are strongly regular graphs. Then $b_1 = rc_2$, $b_2 = a_3 + 1$, $a_2 = (r - 1)(c_2 + 1)$, $c_3 = r(c_2 + 1)$, $a_1 = a_3 + r - 1$, $k_2 = kr$, $k_3 = k(a_3 + 1)/(c_2 + 1)$, $p^1_{33} = a_3(a_3 + 1)/(c_2 + 1) = \mu(\Gamma_3)$, and the intersection array of Γ is $\{r(c_2 + 1) + a_3, rc_2, a_3 + 1; 1, c_2, r(c_2 + 1)\}$.*

Theorem 2. *A distance-regular graph with intersection array $\{44, 35, 3; 1, 5, 42\}$ or $\{48, 35, 9; 1, 7, 40\}$ do not exist.*

Theorem 3. *Let* Γ *be a primitive distance-regular graph of diameter* 3*, where* Γ_2 *and* Γ_3 *are strongly regular graphs. If* Γ_3 *is triangle-free and* $\mu(\Gamma_3) \leq 11$ *then the intersection array of* Γ *either is equal to* $\{119, 100, 15; 1, 20, 105\}$ *or belongs to the following finite series* $\{(r+5)((r+3)^2-3)/6, r(r+3)(r+8)/6, r+6; 1, (r+3)(r+8)/6, r(r+5)(r+6)/6\}$*, where* $r \in \{4, 6, 10, 16, 19, 24, 28, 40, 46, 52, 58, 60, 70, 79\}$.

This part of our work was supported by the grant of Russian Science Foundation, project no. 15-11-10025.

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Manifolds and elements of Catastrophe theory

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The main goal of this talk is to present an elementary introduction to the Catastrophe theory and describe an example of its application to medical problems.

Why is the hyperbolic metrics better than the Euclidean one?

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We describe W. Thurston method for constructing hyperbolic 3-manifolds and present a test for hyperbolicity.

Jacobians of circulant graphs and their generalisations

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The notion of the Jacobian group of a graph, which is also known as the Picard group, the critical group, and the dollar or sandpile group, was independently introduced by many authors ([1–5]). We define Jacobian of a graph as the maximal Abelian group generated by the flows obeying two Kirchhoff's laws. This notion arises as a discrete version of the Jacobian in the classical theory of Riemann surfaces. It also admits a natural interpretation in various areas of physics, coding theory, and financial mathematics. The Jacobian group is an important algebraic invariant of a finite graph. In particular, its order coincides with the number of spanning trees of the graph, which is well known for some simplest graphs, such as the wheel, fan, prism, ladder, and Möbius ladder [6]. At the same time, the structure of the Jacobian is known only in particular cases ([1–3]).

The class of circulant graphs is fairly large and includes the cyclic graphs, complete graphs, Möbius ladder, antiprisms, and other graphs. The purpose of this report is to determine the structure of the Jacobian for circulant graphs, the generalized Petersen graph, I-,Y-,H- graphs and their generalizations. We also present new formulas for the number of spanning trees and investigate arithmetical properties of these numbers. In many important cases, we describe the Jacobian group explicitly. In the general case, we propose an effective algorithm for its calculation.

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A matrix approach to Yang multiplication

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A quadruple of (± 1)-sequences $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d})$ of length m, m, n, n, respectively, is called *base sequences* if $N_{\boldsymbol{a}}(j) + N_{\boldsymbol{b}}(j) + N_{\boldsymbol{c}}(j) + N_{\boldsymbol{d}}(j) = 0$ for all positive integers j, where $N_{\boldsymbol{s}}(j) = \sum_{i=0}^{l-j-1} s_i s_{i+j}$ if $0 \leq j < l$, 0 otherwise, for $\boldsymbol{s} = (s_0, \ldots, s_{l-1}) \in \{\pm 1\}^l$. We denote by $BS(m,n)$ the set of base sequences of length m, m, n, n. Yang proved in [3] that, if $BS(m+1,m) \neq \emptyset$ and $BS(n+1,n) \neq \emptyset$, then $BS(m',m') \neq \emptyset$ with $m' = (2m+1)(2n+1)$.

The well-known Hadamard conjecture states that Hadamard matrices of order $4n$ exist for every positive integer n. The above theorem could be used to settle the restricted version of Hadamard conjecture which claims Hadamard matrices of order $8m$ exist for every odd integer m. Indeed, a class of sequences called T-sequences with length $2m'$ can be obtained from $BS(m',m')$ and Hadamard matrices of order $8m'$ can be produced from T-sequences with length $2m'$ by using Goethals–Seidel arrays.

In the first talk, we use two-variable Laurent polynomials attached to matrices to encode properties of compositions of sequences. The Lagrange identity in the ring of Laurent polynomials is then used to give a short and transparent proof of the above theorem.

In the second talk, we present a generalization of another result of Yang [2] about the construction of paired ternary sequences from base sequences. Here we use a modified version of the Lagrange identity in the ring of Laurent polynomials, together with the matrix approach developed in the first talk.

This is based on joint work with Pritta Etriana Putri.

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Pronormality of subgroups in finite groups

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According to P.Hall, a subgroup H of a group G is said to be *pronormal*, if H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in G$. The following subgroups are pronormal in every finite group: normal subgroups, maximal subgroups, and Sylow p-subgroups. In terms of pronormality one can formulate some properties of groups which are used in algebraic combinatorics. For example, a finite group G is a CI-group if and only if $\mathrm{Sym}_{|G|}$ contains a pronormal regular subgroup isomorphic to G.

In the talk, we discuss problems of pronormality for some special classes of subgroups of finite groups: Hall subgroups, subgroups of odd indices in simple groups, maximal π-subgroups and submaximal π-subgroups in minimal non-solvable groups etc. The talk is based on joint works with W. Guo, A. Kondratiev, N. Maslova, M. Nesterov and E. Vdovin.

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Non-existence of some strongly regular graphs via the unit vector representation

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A regular graph of valency k with v vertices is called *strongly regular* if any two adjacent vertices have λ common neighbours and any two non-adjacent vertices have μ common neighbours. We write $srg(v, k, \lambda, \mu)$ for any graph satisfying these conditions.

In [2], Haemers proved non-existence of $srg(76, 21, 2, 7)$. He used clever edge counting to establish that the neighbourhood of each vertex is a union of cliques. This means that the graph in question is the collinearity graph of a generalized quadrangle of order $(3, 6)$, which does not exist due to an earlier result of Dixmier and Zara.

It is well-known that every distance-regular graph can be represented by a set of unit vectors in an eigenspace of the adjacency matrix of the graph. In this representation, the angle between two vectors depends only on the distance between the corresponding vertices. This representation was utilized, in particular, by Ivanov and the speaker to characterize certain classes of distance-regular graphs. In a joint project with M.R. Alfuraidan and I.O. Sarumi, we are trying to see whether the same ideas work equally well for strongly regular graphs. As a taster case, we obtained a new proof of Haemers' theorem. The key idea is that the Gram matrix corresponding to any set of vertices must be positive semidefinite. It is interesting that root systems arise twice in our proof.

The general structure of the proof is similar to Haemers'. We also aim to show that the graph is locally a union of cliques. Once this is achieved, we do not stop, though, but instead use our unit vector setup to obtain an outright contradiction. Thus, we also provide an alternative proof of Dixmier and Zara's result.

Towards the end of the talk we will discuss how the same ideas can be utilized in one of the open cases of strongly regular graphs.

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Symmetrical extensions of graphs

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A connected locally finite graph Γ is called a symmetrical extension of a graph Γ_1 by a graph Γ_2 if there exist a vertex-transitive group G of automorphisms of Γ and an imprimitivity system σ of G such that the quotient graph Γ/σ is isomorphic to Γ_1 and the blocks of σ are isomorphic to Γ_2. Symmetrical extensions of graphs are of interest for group theory and graph theory. In the talks, we mostly consider the case when Γ_1 is a grid and Γ_2 is a finite graph. This case is also of interest for crystallography and physics.

On the k-closure of a finite permutation group

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Let Ω be a finite set and $G \leq \mathrm{Sym}(\Omega)$. Given a positive integer k, the action of G on Ω induces the componentwise action on Ω^k: $(\alpha_1, \dots, \alpha_k)^g = (\alpha_1^g, \dots, \alpha_k^g)$ for $\alpha_1, \dots, \alpha_k \in \Omega$ and $g \in G$. The orbits of the induced action are called *k-orbits* of G. The largest subgroup of $\mathrm{Sym}(\Omega)$ with the same k-orbits as G is called the *k-closure* of G and denoted by $G^{(k)}$ (see [1]). We are going to discuss some aspects of the following general

k-closure problem. Given a permutation group G, find $G^{(k)}$.

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Algebraic properties of monoids of diagrams and 2-cobordisms

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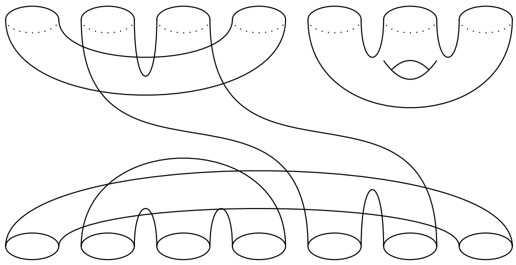
This is a joint work with Karl Auinger, Yuzhu Chen, Xun Hu, and Yanfeng Luo.

A 2-cobordism of degree n is a compact 2-dimensional manifold having $2n$ boundary components. The picture below shows a 2-cobordism of degree 7.



All 2-cobordisms of a given degree form a monoid under a natural composition. This monoid turns out to be an interesting algebraic object with a rich structure, and we explore its semigroup-theoretic properties. One of the main results is that for each $n \geq 1$, the identities holding in the monoid of all 2-cobordisms of degree n do not follow from any finite set of axioms.

Lipschitz polytopes of metric spaces

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Let X be a finite set. For any map c from $X \times X$ to $\mathbb{R}$, the polytrope of c is the set $\{f \in \mathbb{R}^X : f(u) - f(v) \leq c(u,v), \forall u, v \in X\}$, which is an important concept in the study of tropical convexity [2]. When (X, c) forms a metric space, the intersection of the corresponding polytrope with the hyperplane $\{f \in \mathbb{R}^X : \sum_{x \in X} f(x) = 0\}$ is called the Lipschitz polytope of (X, c) and its polar is essentially the fundamental polytope of (X, c). Vershik suggested to study the combinatorics of the fundamental polytopes of finite metric spaces [1], which is equivalent to the study of Lipschitz polytopes.

In this talk, I will report my recent joint work with Zeying Xu [3, 4] on the Lipschitz polytopes of metric spaces and tree metrics. Our work is a small step in the bigger project of understanding general weighted vector configurations [1].

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Contributed Talks

Geometrical structures on the figure-eight knot with a bridge

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A Euclidean structure on the figure-eight knot 4_1 arises when its conical angle α equals 2π; this is due to Thurston [1]. An explicit construction of fundamental set for a cone-manifold $4_1(\alpha)$ in E^3 was given in [2]. The existence of the euclidean structure on figure-eight with a bridge was shown in [3].

In the present work we consider a two-parameter family of cone manifolds $4_1(\alpha, \gamma)$ which singular set is the figure-eight knot with a bridge with conical angles α and γ along them. For such cone manifolds we construct a fundamental set. That is a non-convex polyhedron P having 22 triangular faces and 12 vertices embedded into the Cayley-Klein model of H^3. We establish existence conditions for the hyperbolic structure on $4_1(\alpha, \gamma)$. The domain of existence of such manifolds is bordered by following curves

$$(1)\quad 5\,Y^2(1-X)^2 + Y X(4\,X-5) + X(3-X) + Y = 1,$$
$$(2)\quad 2\,Y^2(1-X) + 2\,X = 1,$$
$$(3)\quad 10\,Y^2(1-X) + 2\,Y(1-4\,X) = 5,$$

where $X = \cos\alpha,\ Y = \cos\theta$ and θ is the angle between two opposite edges of P forming the knot 4_1 as the component of the singular set.

The curve (1) corresponds to the existence of the Euclidean structure on $4_1(\alpha, \gamma)$ which is confirmed by the results of [3]. The curve (3) corresponds to the case $\gamma = 2\pi$ when the bridge disappears as the component of the singular set. Thus (3) defines the set of hyperbolic cone manifolds $4_1(\alpha)$ which is coincide with [2].

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On support sets of acyclic digraphs

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According to [1–3] every binary relation $\sigma \subseteq X^2$ (where X is an arbitrary set) generates a characteristic function on the set X^2 (if $(x, y) \in \sigma$, then $\sigma(x, y) = 1$, otherwise $\sigma(x, y) = 0$). In terms of characteristic functions on the set of all binary relations of the set X, the concept of a binary reflexive adjacency relation is introduced and an algebraic system $G(X)$ consisting of all binary relations of the set X and of all the unordered pairs of adjacent binary relations is defined. If $\operatorname{card} X < \infty$, then $G(X)$ is a graph (the «graph of digraphs»). In the general case, we also call an algebraic system $G(X)$ by a graph. The diameter of the non-trivial graph $G(X)$ is 2.

Further we set $\operatorname{card} X < \infty$ and identify the binary relations and the corresponding digraphs. The following assertions are true. If one of the adjacent digraphs of the graph $G(X)$ is acyclic, then the other digraph is acyclic. If one of the adjacent digraphs of the graph $G(X)$ is transitive, then the other digraph is transitive. (It is well known that every transitive digraph is acyclic.)

For any acyclic digraph σ there is defined a connected component $G_\sigma(X)$ of the graph $G(X)$ containing relation σ, there are defined non-empty support sets

$$S(\sigma) \doteq \{\, y \in X \colon\ \sigma(x, y) = 0 \ \text{ for all } \ x \in X \,\},$$

$$S'(\sigma) \doteq \{\, x \in X \colon\ \sigma(x, y) = 0 \ \text{ for all } \ y \in X \,\},$$

there are defined the families

$$S(G_\sigma) \doteq \{\, S(\tau) \subseteq X \colon\ \tau \in G_\sigma(X) \,\},$$
$$S'(G_\sigma) \doteq \{\, S'(\tau) \subseteq X \colon\ \tau \in G_\sigma(X) \,\},$$

consisting of all support sets of acyclic digraphs τ included in the component $G_\sigma(X)$.

Theorem. *Equality $S(G_\sigma) = S'(G_\sigma)$ is valid.*

The family $S(G_\sigma)$ $\big(= S'(G_\sigma)\big)$ is a specific partially ordered set with respect to the natural relation of inclusion of sets. Specificity is that, together with each element, the family $S(G_\sigma)$ contains all non-empty subsets of this element, and, in addition, $S(G_\sigma)$ contains all singleton subsets of the set X. Moreover, if σ is a transitive digraph, then the family $S(G_\sigma)$ contains all two-element subsets of the set X. The latter circumstance can play an important role in the process of separating transitive digraphs from acyclic digraphs. In connection with this fact, we consider the central problem of an independent description of families $S(G_\sigma)$ (or their maximal elements).

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On finite non-solvable groups whose the prime graphs contain no triangles

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Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G. The prime graph (or the Gruenberg-Kegel graph) of G is the graph whose the vertex set is $\pi(G)$, and two vertices p and q are adjacent if and only if G contains an element of order pq.

The authors investigate the problem of the description of the structure of a finite group G whose the prime graph contains no triangles (3-cycles). It is easy to see that the quotient $G/S(G)$ of G by the solvable radical $S(G)$ is almost simple.

In [1], we found the isomorphic types of prime graphs and estimates of the Fitting length for solvable groups G and determined almost simple groups G.

In [2], we proved that $|\pi(G)| \leq 8$ and $|\pi(S(G))| \leq 3$ for non-solvable groups G. Moreover, a detailed description of the structure of a non-solvable group G in the case when $\pi(S(G))$ contains a number not dividing the order of the group $G/S(G)$ (if $|\pi(S(G))| = 3$, then it is always so).

Now we continue to study the structure of non-solvable groups G. Using the previous results, we can suppose that $1 \leq |\pi(S(G))| \leq 2$ and $\pi(S(G)) \subseteq \pi(G/S(G))$.

In the given work, we proved the following two theorems.

Theorem 1. *Let G be a finite non-solvable group whose prime graph contains no triangles. If $S(G) = O_r(G) \neq 1$ for some prime divisor $r > 3$ of the order of $G/S(G)$ then either G contains an element of order 6 and $G/S(G)$ is isomorphic to $Aut(Sz(8))$ for $r \in \{7, 13\}$, or G contains no elements of order 6.*

Theorem 2. *Let G be a finite non-solvable group whose prime graph contains no triangles. If $|\pi(S(G))| = 2$ then $\pi(S(G)) = \{2, p\}$ for an odd prime p and the vertices 2 and p are adjacent in the graph $\Gamma(G)$; in particular, for any $r \in \pi(G) \setminus \{2, p\}$, Sylow r-subgroups of G are cyclic.*

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Automorphism Group of biquartic Graphs

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A simple graph Γ is called **bipartite** if its vertex set can be partitioned into two subsets A and B of $V(\Gamma)$ in such a way that each edge of Γ connect a vertex in A and a vertex in B. Γ is said to be **biquartic** if it is quartic and bipartite.

In this talk, we report our recent results on the automorphism group of biquartic graphs. Some open questions are also presented.

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Cross-connections of completely 0-simple semigroups

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Cross-connection theory propounded by K. S. S. Nambooripad describes the ideal structure of a (regular) semigroup using the categories of principal left (right) ideals. A semigroup is called completely 0-simple if it is 0-simple and contains a primitive idempotent.

In this talk, we discuss the cross-connection structure of a completely 0-simple semigroup. We characterise the categories of principal left (right) ideals in terms of sets and groups. We describe the construction of certain intermediary completely 0-simple semigroups which arises as Rees quotients of semigroup wreath products. Further, we observe that every cross-connection between these categories is determined by a matrix of group elements and show that this matrix is nothing but the sandwich matrix of the semigroup. This leads to the cross-connection representation of a completely 0-simple semigroup.

On threshold graphs and realizations of graphical partitions

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A *partition* [1] of an integer n is a sequence of nonnegative integers in nonincreasing order which sum is equal to n. The length of a partition is the number of its nonzero parts. A *graphical partition* λ is a partition which nonzero parts can be interpreted as the degree sequence of some simple (undirected) graph G; any such a graph G is called the *realization* of a partition λ. The set of all graphical partitions of $2m$, for a given m, is an order ideal of the lattice of all partitions of $2m$ (see [2] and [3]) ordered by dominance.

Let (x, v, y) be a triple of vertices in a graph $G = (V, E)$ such that $xv \in E$ and $vy \notin E$. The triple is called *lifting* if $\deg(x) \leq \deg(y)$ and *lowering* if $\deg(x) \geq 2 + \deg(y)$. A transformation of the graph G that replaces the edge xv with the edge vy is called *lifting* (*lowering*) *rotation* of an edge if (x, v, y) is a lifting (lowering) triple.

We find a new criterion for a graph to be a threshold graph [4]. Our proof of this criterion is not needed for any another criteria.

Theorem. *A graph G is a threshold graph if and only if it has no lifting triples of vertices.*

This result has three direct corollaries.

Corollary 1. *The graphical partition corresponding to a graph G is a maximal graphical partition if and only if G is a threshold graph (see [4]).*

Corollary 2. *An arbitrary partition is a maximal graphical partition if and only if its head is equal to its tail [5].*

Corollary 3. *Every realization of an arbitrary graphical partition μ can be obtained by a finite sequence of lowering rotations of edges from a threshold realization of an appropriate maximal graphical partition λ such that $\lambda \geq \mu$. Every graph can be transformed to some threshold graph by a finite sequence of lifting rotations of edges.*

Corollaries 1 and 2 give us opportunity to obtain a new proof of Kohnert's criterion [6] for graphical partitions. It is easy to see that all another well known criteria for a partition to be graphical can be obtained from Kohnert's criterion (see [4] and [7]).

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Finite groups with four classes of conjugate maximal subgroups

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Finite groups having the only class of conjugate maximal subgroups are evidently primary cyclic. Finite groups having exactly two classes of conjugate maximal subgroups where described in 1964 by G. Pazderski [1]; in particular, these groups are biprimary. The description of finite groups having exactly three classes of conjugate maximal subgroups was obtained by the author [2] in 1986. In particular, a nonsolvable group G have exactly three classes of conjugate maximal subgroups if and only if $G/\Phi(G)$ is isomorphic to either $PSL_2(7)$ or $PSL_2(2^r)$, where r is a prime. Solvable groups with exactly three classes of conjugate maximal subgroups were completely described in [2].

Now the author investigates finite groups with exactly four classes of conjugate maximal subgroups. We adduce here one of obtained results which give the classification of the all simple groups with this property.

Theorem 1. *A finite simple group G has exactly four classes of conjugate maximal subgroups if and only if one of the following conditions holds:*

(1) $G \cong A_7$;
(2) $G \cong PSL_2(11)$;
(3) $G \cong PSL_2(p)$, *where p is a prime, $p > 3$, and $p \equiv \pm 3, \pm 13 \pmod{40}$;*
(4) $G \cong PSL_2(p^{r^m})$, *where p and r are primes, if $r > 2$ then $p > 2$, $m \in \mathbb{N}$, and $pm > 2$;*
(5) $G \cong PSL_3(3)$;
(6) $G \cong PSU_3(q)$, *where either $q = 3$ or $q = 2^{2^m}$ with $m \in \mathbb{N}$;*
(7) $G \cong Sz(2^r)$, *where r is an odd prime.*

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Codes in Shilla graphs with $b_2 = c_2$

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We consider undirected graphs without loops and multiple edges. For a vertex a of a graph Γ the subgraph $\Gamma_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. Let $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

Degree of a vertex a in Γ is the number of vertices in $[a]$. A graph Γ is called regular of degree k if degree of any vertex is equal to k. A graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, $|[u] \cap [w]|$ is equal to λ if u adjacent to w and is equal to μ if $d(u,w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Let Γ be a distance-regular graph of diameter $d = 2e + 1$, e be a positive integer, and C be an e-code in Γ. Then $|C| \leq p^d_{dd} + 2$. If the equality is achieved then C is called a maximal e-code.

Let C be a maximal e-code in Γ. Then $c_d \geq a_d p^d_{dd}$. If the equality is achieved then C is called a locally regular e-code.

Let C be an e-code in Γ. Then $|C| \leq k_d / \sum_{i=0}^{e} p^d_{id} + 1$. If the equality is achieved then C is called a last subconstituent perfect e-code.

In [1, Proposition 5] the following proposition was proved.

Proposition 1. *Let Γ be a distance-regular graph of diameter 3 containing a maximal 1-code C that is both locally regular and last subconstituent perfect. Then the intersection array of Γ is either $\{a(p+1), cp, a+1; 1, c, ap\}$ or $\{a(p+1), (a+1)p, c; 1, c, ap\}$, where $a = a_3$, $c = c_2$, and $p = p^3_{33}$.*

In the last case Γ is a Shilla graph. Then a divides k and let $b = b(\Gamma) = k/a$. In this work we prove the following theorems.

Theorem 1. *Let Γ be a Shilla graph with $b_2 = c_2$ and $\theta_2 = 0$. Then $b_2 = bs$, $a = (b+1)s$, $\theta_3 = s - b - bs$ either $2s + 1 \leq b - 2$ or either $2s + 1 = b$ and the intersection array of Γ is $\{2s(s+1)(2s+1), 2s(s+1)^2, s(2s+1); 1, s(2s+1), 4s^2(s+1)\}$, or $2s + 1 = b + 1 = 3$ and the intersection array of Γ is $\{6, 4, 2; 1, 2, 3\}$.*

Theorem 2. *Let Γ be a Shilla graph with $b_2 = c_2$ and $\theta_2 = 1$. Then either $a-1 = t(b+3)$, $c_2 = t(b+1)$ and if $t \leq 3$ then the intersection array of Γ is $\{225, 208, 20; 1, 20, 200\}$, or $b+3$ does not divide $a-1$, $2(a-1) = t(b+3)$, $c_2 = t(b+1)/2$ and if $t \leq 3$ then the intersection array of Γ is $\{12, 10, 2; 1, 2, 8\}$.*

Theorem 3. *Let Γ be a Shilla graph with $b_2 = c_2$ and $b = 5$. Then the intersection array of Γ is one of the following: $\{25, 24, 3; 1, 3, 20\}$, $\{30, 28, 2; 1, 2, 24\}$, $\{35, 32, 8; 1, 8, 28\}$, $\{50, 44, 5; 1, 5, 40\}$, $\{60, 52, 10; 1, 10, 48\}$, $\{65, 56, 5; 1, 5, 52\}$, $\{75, 64, 8; 1, 8, 60\}$, $\{135, 112, 12; 1, 12, 108\}$.*

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Q-polynomial Shilla graphs with $b_2 = c_2$

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In [1, Proposition 5] the following proposition was proved.

Proposition 1. *Let Γ be a distance-regular graph of diameter 3 containing a maximal 1-code C that is both locally regular and last subconstituent perfect. Then the intersection array of Γ is either $\{a(p+1), cp, a+1; 1, c, ap\}$ or $\{a(p+1), (a+1)p, c; 1, c, ap\}$, where $a = a_3$, $c = c_2$, and $p = p^3_{33}$.*

In the last case Γ is a Shilla graph with $b_2 = c_2$. Then $a = a_3$ divides k and for $b = b(\Gamma) = k/a$ we have $p^3_{33} = b-1$. In this work exact formulas for multiplicities eigenvalues of Shilla graphs with $b_2 = c_2$ were founded.
A Shilla graph with $b_2 = c_2$ is Q-polynomial if and only if $\theta_3 = -b(b+1)/2$. In this case $b = 2r$, $c_2 = (t+r)r$, and the intersection array of Γ is $\{2rt(2r+1), (2r-1)(2rt+t+1), r(r+t); 1, r(r+t), t(4r^2-1)\}$. Moreover, in view of [2, Proposition 4.6.3] for any vertex $u \in \Gamma$ the subgraph $\Gamma_3(u)$ is an antipodal distance-regular graph with the intersection array $\{t(2r+1), (2r-1)(t+1), 1; 1, t+1, t(2r+1)\}$ and $t \le 2r(r+1)(2r-1) - r$. Note that in the case $t = r$ we have $r = 1$.
In this work by using the exact formulas for multiplicities of eigenvalues a new infinite series of feasible intersection arrays of hypothetic Q-polynomial Shilla graphs with $b_2 = c_2$ was founded.

Proposition 2. *A Shilla graph with the intersection array $\{2rt(2r+1), (2r-1)(2rt+t+1), r(r+t); 1, r(r+t), t(4r^2-1)\}$ has following multiplicities of nonprincipal eigenvalues:*
$2(4r^2t + 2rt + r - t)r/(r + t)$,
$2(4r^2t + 2rt + r - t)(2rt + t + 1)t/((2r^2 + t)(r + t))$,
$2(2rt + t + 1)(2r + 1)^2(2r - 1)rt/((2r^2 + t)(r + t))$.

Theorem 1. *Let Γ be a Q-polynomial Shilla graph with $b_2 = c_2$. Then an intersection array $\{2r(2r^2-1)(2r+1), (2r-1)(2r(2r^2-1)+2r^2), r(2r^2+r-1); 1, r(2r^2+r-1), (2r^2-1)(4r^2-1)\}$ for Γ and an intersection array $\{(2r^2-1)(2r+1), 2r^2(2r-1), 1; 1, 2r^2, (2r^2-1)(2r+1)\}$ for $\Gamma_3(u)$ are feasible.*

Problem 1. *What is the automorphism group of distance-regular graph with the following intersection array $\{2r(2r^2-1)(2r+1), (2r-1)(2r(2r^2-1)+2r^2), r(2r^2+r-1); 1, r(2r^2+r-1), (2r^2-1)(4r^2-1)\}$?*

This work was supported by the grant of Russian Science Foundation, project no. 14-11-00061-P.

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On a structure of locally finite subgroups of a finitary linear group over a commutative Noetherian ring

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Let $FL_{\nu}(K)$ be a finitary linear group, where K is a ring with unit and let ν be a linearly ordered set. $FL_{\nu}(K)$ is investigated in [1], [2]. In particular, the finitary unitriangular group $UT_{\nu}(K)$ is studied in [2]. We studied periodic subgroups of $FL_{\nu}(K)$ where K is a Dedekind ring [3].

The main result of this talk is the following theorem.

Theorem. If G is a periodic subgroup of $FL_{\nu}(K)$, where K is a commutative Noetherian ring then G is locally finite. If ν is countable then $G = \cup_{i\in\mathbb{N}} G_i$, where $G_1 \leq G_2 \leq \cdots \leq G_i \leq \cdots$ and the following conditions hold:

(1) G_i has a series of normal subgroups $A_i \leq K_i \leq N_i \leq G_i$, where A_i is Abelian, K_i/A_i and N_i/K_i are nilpotent and G_i/N_i is countable for any $i \in \mathbb{N}$;

(2) $N_1 N_2 \cdots N_i \cdots$ is a subgroup of G;

(3) $N_1 N_2 \cdots N_i/N_i$ is countable for any $i \in \mathbb{N}$.

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Full and elementary nets over the field of fractions of a principal ideal ring

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Let R be a commutative ring with unit and n be an integer. The set $\sigma = (\sigma_{ij}), \quad 1 \leq i, j \leq n$, of additive subgroups of the ring R is called a *net (carpet)* under the ring R of the order n, if $\sigma_{ir}\sigma_{rj} \subseteq \sigma_{ij}$ for all values of the indices i, r, j.

An *elementary net* of order n over R [1–3] is a set of additive subgroups (without diagonal) $\sigma = (\sigma_{ij}), \quad 1 \leq i \neq j \leq n$, of R for which

$$\sigma_{ir}\sigma_{rj} \subseteq \sigma_{ij}, \quad i \neq j, i \neq r, r \neq j, \quad 1 \leq i, r, j.$$

A net is called *irreducible*, if $\sigma_{ij} \neq 0$ for all i, j. An example of an irreducible net is the net of *constant* σ_P, defined for an arbitrary non-zero ring P such that $(\sigma_P)_{ij} = P$ for all i, j. Let K be a field of fractions of a principal ideal ring R, and $\sigma = (\sigma_{ij})$ be a full (elementary) net of order $n \geq 2$ (respectively, $n \geq 3$) over K such that the additive subgroups σ_{ij} are nonzero R-modules. It is proved that, up to conjugation by diagonal matrix, all σ_{ij} are ideals of a fixed intermediate subring P, $R \subseteq P \subseteq K$.

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Relative resolvability of topological spaces

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The notion of the relative resolvability was introduced by N.V. Velichko [1]. A topological space X is called *relative resolvable* if there exists a two-valued real function defined on X whose points of continuity are isolated in X. This definition is closely related to the notion of resolvability of topological spaces, which was introduced by Hewitt [2].

The resolvability (ω-resolvability) of Lindelöf spaces whose dispersion character is uncountable was proved in [3] and [4].

In this talk, we will consider generalizations of Lindelöf spaces and prove their relative resolvability under additional (natural) assumptions. The property of relative resolvability of certain topological spaces, in particular, of linearly Lindelöf spaces, will also be discussed.

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On chromaticity uniqueness of some full tripartite graphs

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Let G be a simple finite graph and $P(G, x)$ be its chromatic polynomial. Two graphs G and H are called *chromatically equivalent* if $P(G, x) = P(H, x)$ for all x. A graph G is called *chromatically unique* if for every graph H such as $P(G, x) = P(H, x)$ implies that graphs G and H are isomorphic.

In [2–4] chromatic uniqueness is proved for all complete tripartite graphs $K(n_1, n_2, n_3)$ such as $n_1 \geq n_2 \geq n_3 \geq 2$ and $n_1 - n_3 \leq 4$.

The main result of this paper is the following theorem.

Theorem. A complete tripartite graph $K(n_1, n_2, n_3)$, where $n_1 \geq n_2 \geq n_3 \geq 2$, $n_1 - n_3 \leq 5$ and $n_1 + n_2 + n_3 \not\equiv 2 \pmod 3$ is chromatically unique.

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Grundy dominating sequences and zero forcing sets

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In a graph G a sequence $v_1, v_2, \ldots, v_m$ of vertices is a legal dominating sequence if for all $2 \leq i \leq m$ we have $N[v_i] \not\subseteq \cup_{j=1}^{i-1} N[v_j]$. The maximum length of a legal dominating sequence in G is the Grundy domination number of a graph G and is denoted by $\gamma_{gr}(G)$. In the talk the exact values of Grundy domination number will be presented for some families of graphs [1, 2] and a strong connection to the zero forcing number of a graph G will be established [3]. The latter invariant is closely related to the minimum rank of a graph G, $\mathrm{mr}(G)$, which is the smallest possible rank over all symmetric real matrices whose (i, j)-th entry, for $i \neq j$, is nonzero whenever vertices i and j are adjacent in G and is zero otherwise.

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On Eigenfunctions and Maximal Cliques of Paley Graphs of Order q^2

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This is joint work with V. Kabanov, L. Shalaginov and A. Valyuzhenich

Many combinatorial configurations (perfect codes, latin squares and hypercubes, combinatorial designs) can be viewed as an eigenfunction on a graph with some discrete restrictions. The study of these configurations often leads to the question about the minimum possible difference between two configurations from the same class (it is often related with bounds of the number of different configurations; for example, see [1–5]). Since the symmetric difference of these two configurations is also an eigenfunction, this question is directly related to the minimum cardinality of the support (the set of nonzero) of an eigenfunction with given eigenvalue. This talk is devoted to the problem of finding the minimum cardinality of the support of eigenfunctions in the Paley graphs of order q^2 (denote it by $P(q^2)$), where q is prime power. Currently, a similar problem is solved for Hamming graphs $H(n,q)$ for $q = 2$ (see [4]). In [7] Vorob'ev and Krotov proved the lower bound on the cardinality of the support of an eigenfunction of the Hamming graph. In [6] the minimum cardinality of the support of eigenfunctions in the Hamming graphs with the second largest eigenvalue $n(q-1)-q$ was found.

In this talk, we present construction of eigenfuctions on $P(q^2)$ for both non-principal eigenvalues; the cardinality of the support of the eigenfunctions is equal to $q+1$. It follows from [3], that our construction gives eigenfunctions with minimum cardinality of support. As a related topic, we discuss maximal cliques in $P(q^2)$ of order $(q+1)/2$ and $(q+3)/2$, where $q \equiv 1(4)$ and $q \equiv 3(4)$, correspondingly. In [8], Baker, Ebert, Hemmeter and Woldar proposed construction of such cliques, but they noted that their construction doesn't cover all known cliques.

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On Perfect 2-colorings of Johnson Graphs $J(n, 3)$ and Bilinear Forms Graphs $Bil_2(2 \times d)$

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This is joint work with Alexander Gavrilyuk

A perfect t-coloring (or an equitable t-partition) of a graph Γ with t colors is a partition of the vertex set of Γ into t colors (parts) $P_1, \ldots, P_t$ such that, for all $i, j \in \{1, ..., t\}$, every vertex of P_i is adjacent to the same number, namely, p_{ij}, of vertices of P_j. The matrix $\Pi := (p_{ij})_{i,j=1,\ldots,t}$ is called the quotient matrix of the perfect t-coloring. It is well known that every eigenvalue of Π is an eigenvalue of the adjacency matrix of Γ. If Γ is regular with valency k and has distinct eigenvalues $k = \theta_0 \geq \theta_1 \geq \ldots \geq \theta_d$, then θ_0 is an eigenvalue of Π, and the smallest $s \geq 0$ such that θ_{s+1} is an eigenvalue of Π is called the strength of perfect t-coloring.

The Johnson graph $J(n, k)$, $2k \leqslant n$, has as vertices all k-element subsets of an n-element set, with two vertices being adjacent if their intersection has cardinality $k-1$. The bilinear forms graph $Bil_q(e \times d)$ is a graph defined on the set of $(e \times d)$-matrices over the finite field $\mathbb{F}_q$ with two matrices being adjacent if the rank of their difference equals 1. The bilinear forms graph $Bil_q(2 \times 2)$ is a subgraph of the Grassmann graph $J_q(4, 2)$, which is defined on the lines of the projective geometry $PG(3, q)$, with two lines being adjacent if they intersect in a point.

In this work we study perfect 2-colorings of the Johnson graphs $J(n, 3)$ and the bilinear forms graphs $Bil_2(2 \times d)$.

The Johnson graph $J(n, 3)$ has the four distinct eigenvalues given by $\theta_i = (3-i)(n-3-i)-i$, $i = 0, \ldots, 3$. In 2003, Meyerowitz [5] classified all perfect 2-colorings of $J(n, k)$ with strength 0, and in 1994 Martin [4] obtained some partial results on perfect 2-colorings of $J(n, k)$ with strength 1. We determine[1] all perfect 2-colorings of $J(n, 3)$ of strength 1, which completes the research begun in [1]. Note that perfect 2-colorings of $J(n, 3)$ with strength 2 are in a one-to-one correspondence with combinatorial 2-designs with blocks of size 3.

Recall that a Cameron-Liebler line class with parameter x in the projective geometry $PG(3, q)$ is a set of lines that shares precisely x lines with every spread of $PG(3, q)$. Cameron-Liebler line classes induce perfect 2-colorings of the Grassmann graphs $J_q(4, 2)$ and, moreover, perfect 2-colorings of the corresponding bilinear forms graphs $Bil_q(2 \times 2)$ (with strength 0 in both graphs) [2]. Cameron-Liebler line classes seem to be quite rare [3]. On the contrast to that, we show[2] that the bilinear forms graphs $Bil_q(2 \times d)$ admit a lot of perfect 2-colorings with stength 0, and determine all of them for $q = 2$.

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Greedy cycles in the Star graphs

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In this work we investigate greedy cycles in the *Star graph* $S_n = Cay(\mathrm{Sym}_n, t)$, $n \geqslant 4$. The Star graph is the Cayley graph on the symmetric group Sym_n of permutations with the generating set t of all transpositions swapping the 1st and ith elements of a permutation. It is a connected bipartite $(n-1)$–regular graph of order $n!$. Since the graph is bipartite, it does not contain odd cycles but it does contain ℓ–cycles for all even ℓ, where $6 \leqslant \ell \leqslant n!$ (with the sole exception when $\ell = 4$) [1]. Hence, the Star graph S_n is hamiltonian, i.e. it contains a hamiltonian cycle on $n!$ vertices. Its hamiltonicity also follows from [2].

There is a connection between hamiltonicity of graphs and combinatorial Gray codes [3], where a combinatorial Gray code has been introduced as a way of generating combinatorial objects so that successive objects differ in some pre–specified small way. By setting a graph and by describing hamiltonian cycles in this graph, one can refer to Gray codes implicitly. In 2013, it was suggested to use greedy sequences to construct prefix–reversal Gray codes in the Pancake graphs [4].

A *greedy sequence* is defined as the ordered set of generating elements of a Cayley graph. The *greedy hamiltonian cycle* is called a hamiltonian cycle formed by consecutive application of the leftmost suitable element of a greedy sequence. A greedy sequence is called a *greedy subsequence* if it forms a non-hamiltonian cycle, which is called a *greedy cycle*.

In this work we apply greedy approach to constructing greedy cycles in the Star graph.

Theorem. *In the Star graph $S_n = Cay(\mathrm{Sym}_n, t)$, $n \geqslant 4$, any ordered set of mutually different $n-1$ elements from the generating set t is a greedy subsequence which forms a cycle of length $\ell = 2 \cdot 3^{n-2}$. Moreover, any greedy subsequence gives $\frac{n!}{6}$ mutually different greedy ℓ-cycles in the graph.*

This theorem gives us the following results.

Corollary 1. *There are no greedy hamiltonian cycles in the Star graph $S_n = Cay(\mathrm{Sym}_n, t)$ for $n \geqslant 4$.*

Corollary 2. *There is a vertex disjoint cycle cover in the Star graph $S_n = Cay(\mathrm{Sym}_n, t)$, $n \geqslant 4$, presented by greedy cycles of lengths $2 \cdot 3^{k-2}$, where $3 \leqslant k \leqslant n$.*

The proof of the results above is mainly based on the hierarchical structure of the Star graph S_n.

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Zero product determined algebras

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An algebra $\mathcal{A}$ is said to be zero product determined if every bilinear map B from $\mathcal{A} \times \mathcal{A}$ into an arbitrary vector space V with the property that $B(x, y) = 0$ whenever $xy = 0$ is of the form $B(x, y) = f(xy)$ for some linear map $f : \mathcal{A} \to V$. We obtain conditions on a unital algebra containing a nontrivial idempotent making it zero product determined algebra. It is also not hard to see, that every finite dimensional unital algebra generated with its idempotents (i. e. matrix algebra M_n) is zero product determined.

We describe the form of generalized derivations determined by action on zero product elements on some special classes of unital algebras containing a nontrivial idempotent. For example, if $\mathcal{A}$ is zero product determined algebra every generalized derivation on $\mathcal{A}$ determined by action on zero products is of the standard form. In general there can also exist nonstandard solutions.

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On the largest element orders of the simple orthogonal groups of characteristic 2

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This talk is based on joint work with Daniel Lytkin. We were motivated by the paper [1] where Kantor and Seress proved that the three largest element orders determine the characteristic of a simple group of Lie type of odd characteristic and, as a part of the proof, found the two largest element orders in all simple groups of Lie type of odd characteristic. The obstacles for finding the largest orders in characteristic 2 come from orthogonal and symplectic groups and are related to the fact that there are arbitrarily large collections of pairwise relatively prime integers of the form $2^m \pm 1$ (see [1, p. 808] for details).

Recall that the simple symplectic and orthogonal groups in characteristic 2 are exactly $Sp_{2n}(2^m)$, where $n \geqslant 2$ and $(n, m) \neq (2, 1)$, and $\Omega^{\pm}_{2n}(2^m)$ with $n \geqslant 4$. The largest element order of the symplectic group $S = Sp_{2n}(2^m)$ was found independently by Lytkin [2] and Spiga [3]. In addition to $o_1(S)$, Lytkin found $o_2(S)$ and Spiga found $o_1(\operatorname{Aut} S)$ (we write $o_1(G)$ and $o_2(G)$, with $o_1(G) > o_2(G)$, for the two largest orders of elements of a finite group G).

Our main result is the exact values of $o_1(S)$ and $o_2(S)$ for the orthogonal group $S = \Omega^{\varepsilon}_{2n}(2^m)$, where $\varepsilon \in \{+, -\}$, $n \geqslant 4$ and $m > 1$. In particular, we show that these numbers are odd and if $S \neq \Omega^{+}_{8}(2^m)$, then $o_i(\operatorname{Aut} S) = o_i(S)$ for $i = 1, 2$.

The condition $m > 1$ of the previous paragraph is crucial since $o_1(\Omega^{\varepsilon}_{2n}(2))$ is not always odd. In some cases $o_1(\Omega^{\varepsilon}_{2n}(2))$ can be extracted from the results of [2]: they imply that the sets $\{o_1(Sp_{2n}(2)), o_2(Sp_{2n}(2))\}$ and $\{o_1(\Omega^{+}_{2n}(2)), o_1(\Omega^{-}_{2n}(2))\}$ intersect nontrivially for all $n \geqslant 4$. However, these sets quite rarely coincide, so the problem of determining $o_1(\Omega^{\varepsilon}_{2n}(2))$ is still open.

The obstacle for handling $\operatorname{Aut}(\Omega^{+}_{8}(2^m))$ is triality graph automorphisms, since there are no method to calculate the orders of elements in extensions by these automorphisms. It is worth noting that upper bounds for $o_1(\operatorname{Aut} S)$, where S is a simple classical group, are found in [4, Theorem 2.16].

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On dense subsets and projections of Tychonoff products of spaces

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It is well know that the Tychonoff product of 2^ω many separable spaces is separable.

In Tychonoff products of 2^ω many separable spaces we construct countable dense sets such that the projections of their subsets have certain properties.

We prove that in the Tychonoff product of 2^ω many separable Hausdorff not single point spaces there is a countable set that is dense and contains no nontrivial convergent in the product sequences (such set is sequentially closed in the product).

The existence of such set in the Tychonoff product of closed unit intervals was proved by W.H. Priestley.

We prove that in the product of unit intervals there is a countable set, that is dense but sequentially closed in it with the Tychonoff topology of the product and is closed and discrete in it with the box topology.

On the lattice of overcommutative varieties of monoids

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There are many articles devoted to the examination of the lattice of semigroup varieties. An overview of this area is contained in the detailed survey [5]. In sharp contrast, the lattice of monoid varieties has received much less attention over the years (referring to monoid varieties, we consider monoids as algebras with two operations, namely an associative binary operation and the nullary operation wthat fixes the unit element). There are only a few papers devoted to this subject [3, 4, 6]. As a result, many natural questions about the lattice of monoid varieties remain open. In particular, it is unknown so far, whether this lattice satisfies some non-trivial identity. For comparison, we note that the negative answer to the analogous question for semigroup varieties was known since the beginning of 1970's [1, 2].

A variety of monoids is called *overcommutative* if it contains the variety of all commutative monoids. Evidently, the class of all overcommutative varieties forms a sublattice in the lattice of all monoid varieties. We denote this sublattice by **OC**. We prove the following

Theorem. *The lattice* ***OC*** *of all overcommutative monoid varieties does not satisfy any non-trivial identity.*

In actual fact, we verify that the partition lattice over an arbitrary finite set is an anti-homomorphic image of some sublattice of the lattice **OC**. This immediately implies our theorem. The question, whether the lattice **OC** contains an anti-isomorphic copy of arbitrary finite partition lattice still be open.

We note also that the result on the absence of non-trivial identities in the lattice of monoid varieties was recently independently proved in another way by I.A.Mikhaylova (private communication).

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An upper class of CLT-groups

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A finite group with the property that for every divisor of the order of the group there is a subgroup of that order, is called a CLT-group. In this talk, we introduce a class of finite groups that contains all CLT-groups. Now, we call a group G of order n to be a weak CLT-group if for every divisor d of n there exists a subgroup H of G such that $|H| = d$ or $|G : H| = d$. It is clear that the class of these groups contains all CLT-groups. We state some classes of weak CLT-groups and show that the least non weak CLT-group has order 240 (although for CLT-groups it is 12). At last, we pose some interesting problems about the topic.

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On coincidence of classes E_{π_x} and D_{π_x} of finite groups

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Throughout this talk, we will use the term "group" meaning a finite group.

Let π be a set of primes. A π-subgroup H is called a Hall π-subgroup of a group G if the order of H is coprime to the index $|G : H|$. Note that if $\pi = \{p\}$ then a Hall π-subgroup of a group G is exactly a Sylow p-subgroup of G.

Define E_π to be the class of all groups possessing a Hall π-subgroup. Let $C_\pi \subseteq E_\pi$ be the subclass of all groups in which any two Hall π-subgroups are conjugate. And finally, let $D_\pi \subseteq C_\pi$ be the subclass of all groups in which every π-subgroup is contained in some Hall π-subgroup.

In 1956, Ph. Hall conjectured that for any set π of odd primes the classes E_π and D_π coincide. However, Gross [2] disproved the conjecture and showed that for any finite set π of odd primes containing more than one element, the class $E_\pi \setminus D_\pi$ is non-empty. The following problem naturally arises.

Problem. *For which sets π of primes the equality $E_\pi = C_\pi = D_\pi$ holds?*

In the other words, for what sets π does the complete analogue of the Sylow theorems hold for Hall π-subgroups in every group $G \in E_\pi$?

Let x be a real number. Define

$$\pi_x = \{p \mid p \text{ is a prime and } p > x\}.$$

Arad and Ward [1] proved that $E_{\pi_x} = D_{\pi_x}$ for $x < 3$, and Revin [3] proved that $E_{\pi_x} = D_{\pi_x}$ for $x \geq 7$. We prove the following theorem.

Theorem. *For any real number x, the equality $E_{\pi_x} = D_{\pi_x}$ holds.*

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The Security number of the Cartesian product of three paths

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Let G be a simple graph with the set of vertices $V(G)$ and the set of edges $E(G)$. For any vertex v of G we denote *the neighborhood* of v by $N(v) = \{u \,|\, \{u, v\} \in E(G)\}$ and *the closed neighborhood* of v by $N[v] = \{v\} \cup N(v)$. Let X be a subset of $V(G)$. *The open neighborhood* of set X is $N(X) = \bigcup_{v \in X} N(v)$ and *the closed neighborhood* of X is $N[X] = N(X) \cup X$.

Let $S = \{s_1, s_2, \ldots, s_k\}$ be a subset of $V(G)$. For any vertex $s_i \in S$ we denote $A(s_i) = N(s_i) \setminus S$ and $D(s_i) = N[s_i] \cap S$. We say that $s_i \in S$ is *under attack* by $A(s_i)$, and s_i is *defended* by $D(s_i)$. This means that s_i defends itself and its neighbors in S. *An attack on* S are any k mutually disjoint sets $A = \{A_1, A_2, \ldots, A_k\}$ such that $A_i \subseteq A(s_i)$ for $i = 1, 2, \ldots, k$. *A defense* of S are any k mutually disjoint sets $D = \{D_1, D_2, \ldots, D_k\}$ such that $D_i \subseteq D(s_i)$ for $i = 1, 2, \ldots, k$. An attack A on S is said to be *defendable* if there exists a defense D such that $|D_i| \geq |A_i|$ for all $i = 1, 2, \ldots, k$. If any attack A on S is defendable, we call S *a secure set* of G. The *security number* of G, denoted by $s(G)$, is the smallest cardinality of a secure set of G.

The concept of security in graphs was introduced by R.C. Brigham, R.D. Dutton, and S.T. Hedetniemi [1] as a special case of the concept of defensive alliances in graphs [2]. They also gave a characterization of secure sets. In [3] one can find some general lower and upper bounds on the security number.

For graphs G and H, *the Cartesian product* of G and H, is the graph whose vertex set is $\{i, j)| i \in V(G), j \in V(H)\}$ and in which (i, j) is joined to (i_1, j_1) if and only if either $i = i_1$ and $\{j, j_1\} \in E(H)$ or $j = j_1$ and $\{i, i_1\} \in E(G)$. The security number of the Cartesian product of two paths was proved in [1], where also the conjectures for the security number of two-dimensional cylinders $P_m \,\square\, C_n$ and tori $C_m \,\square\, C_n$ were given. Namely, it was stated that $s(P_m \,\square\, P_n) = \min\{m, n, 3\}$, $s(P_m \,\square\, C_n) = \min\{2m, n, 6\}$, $s(C_3 \,\square\, C_3) = 4$, and $s(C_m \,\square\, C_n) = \min\{2m, 2n, 12\}$ for $\max\{m, n\} \geq 4$. This conjectures were later proved by K. Kozawa, Y. Otachi, K. Yamazaki in [4]. They also gave conjecture for the security number of the Cartesian product of three paths.

We investigate the security number of the Cartesian product of three paths. Let $P_l \,\square\, P_m \,\square\, P_n$ be the Cartesian product of three paths. The conjecture in [4] was $s(P_l \,\square\, P_m \,\square\, P_n) \leq \min\{lm, mn, nl, 20\}$. Without loss of generality we can consider $l \leq m \leq n$ because the Cartesian product of graphs is a commutative and associative operation. It is easy to see that $s(P_2 \,\square\, P_2 \,\square\, P_n) = 4$, and $s(P_2 \,\square\, P_m \,\square\, P_n) = 6$. All other cases are covered with the following theorem.

Theorem. *Let $2 < l \leq m \leq n$. If G is a Cartesian product of three paths P_l, P_m, and P_n, then $s(G) = \min\{3l, 17\}$.*

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Quasi-Pyramidal Tours and Polynomial Time Solvable Subclass of the Generalized Traveling Salesman Problem

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The classic Traveling Salesman Problem (TSP) is strongly NP-hard and hardly approximable problem in its general setting. Meanwhile, the problem and its generalizations with additional constraints on feasible routes, e.g. precedence constraints can become tractable [1, 2]. Among others, restricting to so called *pyramidal routes* seem to be the most actively studied (see, e.g. [3]). The route is called pyramidal if it is concordant with the natural order $v_1 < v_2 < \ldots < v_n$ defined on the nodeset of a given graph and has the form

$$v_1 = v_{i_1}, v_{i_2}, \ldots, v_{i_r} = v_n, v_{i_{r+1}}, \ldots, v_{i_n}, \text{ where}$$

$$v_{i_j} < v_{i_{j+1}} \ (1 \leq j \leq r-1), \quad v_{i_j} > v_{i_{j+1}} \ (r+1 \leq j \leq n-1).$$

It is known [4] that optimal pyramidal route can be found by dynamic programming in time $O(n^2)$ for an arbitrary weight function and even in subquadratic time $O(n \log^2 n)$ in the Euclidean setting [5]. Despite their wide familiarity, pyramidal routes can hardly be used for construction of algorithms for general TSP, since instances of this problem having pyramidal route as an optimal or a good suboptimal solution are very rare. Actually, the set of such known instances is exhausted by instances obeying the well-known Demidenko and Van der Veen conditions (see, e.g. [3]).

In this presentation, we consider Generalized Traveling Salesman Problem (GTSP). An instance of GTSP is defined by an edge-weighted (di)graph $G = (V, E)$ and a partition $V_1 \cup \ldots \cup V_k = V$ of its nodeset to k clusters. The goal is to find a minimum weight cycle route visiting each cluster at once.

We introduce a notion of l-quasi-pyramidal route extending the pyramidal routes to the case of GTSP and show that, for any weighting function and any fixed l, an optimal l-quasi-pyramidal route can be found in time $O(4^l n^3)$, i.e. GTSP belongs to the class of FPT problems.

Further, we describe a non-trivial geometric subclass of GTSP, each whose instance has a 20-quasi-pyramidal route as an optimal solution, i.e. it can be solved to optimality in $O(n^3)$.

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Attainable Guarantee for k-medians Problem on $[0,1]$

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We consider the famous k-medians clustering problem, which can be stated as follows. Input: a finite sample $\xi = (x_1, \ldots, x_n)$ from some metric space (X, ρ) and a natural number $k > 1$. It is required to find points $m_1, \ldots, m_k \in X$ such that

$$F(\xi; m_1, \ldots, m_k) = \sum_{i=1}^{n} \min\{\rho(x_i, m_1), \ldots, \rho(x_i, m_k)\} \to \min .$$

Although, the problem is NP-hard even in the Euclidean plane [11], it is easy to verify that, in the one-dimensional space, the problem can be solved to optimality in polynomial time. We consider the case, where all points to be clustered are sampled from $[0,1]$. Our goal is to estimate the least value of F, which can be guaranteed for any n-sample.

These reasonings lead us to zero-sum two-player game, where the first player choose a sample ξ, and the second one partition this sample onto k clusters with centers $m_1, \ldots, m_k$, while the function F is taken as a payoff. It can be shown that, for any $k > 1$, this game has no value, and we are interested in its lower price

$$v_* = \sup_{\xi} \inf_{m_1, \ldots, m_k} F(\xi; m_1, \ldots, m_k).$$

We show, that, for any $k > 1$, $v_* = \frac{n}{2(2k-1)}$, and the bound obtained is attainable. The result presented can be used in development of heuristics for k-medians problem defined in d-dimensional Euclidean spaces.

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On multiplicities of largest eigenvalues of the Star graph

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The Star graph S_n, $n \geqslant 2$, is the Cayley graph on the symmetric group Sym_n with the generating set of all transpositions swapping the 1st and ith elements of a permutation, where $2 \leqslant i \leqslant n$. The spectrum of a graph is the set of eigenvalues of the graph. We consider the set of eigenvalues of the Star graph as the set of eigenvalues of its adjacency matrix. A spectrum is called integral if all eigenvalues of its adjacency matrix are integers. Since the Star graph is bipartite, its spectrum should be symmetric. Since the Star graph is n-regular, its spectrum should lie in the segment $[-(n-1), n-1]$.

In 2009, A. Abdollahi and E. Vatandoost conjectured [1] that the spectrum of S_n is integral, and contains all integers in the range from $-(n-1)$ up to $n-1$ (with the sole exception that when $n \leqslant 3$, zero is not an eigenvalue of S_n). In 2012, R. Krakovski and B. Mohar [5] proved the second part of the conjecture. Moreover, they obtained lower bounds on the multiplicity of eigenvalues. However, the question of the possibility of calculating the exact values of multiplicities of eigenvalues remained open.

In 2012 it was shown by G. Chapuy and V. Feray [3] that the integrality of the Star graph was already solved in another context. It is equivalent to studying the spectrum of so–called Jucys-Murphy elements in the algebra of the symmetric group. This connection between two kinds of spectra implies that the Star graph is integral.

In 2015 this approach was used by E. Konstantinova and the author to obtain the exact values of multiplicities of eigenvalues of the Star graph for $n \leqslant 10$ [4]. Analytic formulas for calculating multiplicities of eigenvalues $\pm(n-t)$ for $t = 2, 3, 4, 5$ of the Star graph were found recently by S. Avgustinovich, E. Konstantinova and the author [2].

In this work we prove the following theorem.

Theorem. *Let $n \geqslant 2$ and $1 \leqslant t \leqslant \frac{n}{2}$, then the multiplicity $mul(n-t)$ of the eigenvalue $(n-t)$ is given by the following formula:*

$$mul(n-t) = \frac{n^{2(t-1)}}{(t-1)!} + P(n),$$

where $P(n)$ is a polynomial of degree less than $2(t-1)$.

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On the realizability of a graph with 6 vertices as the Gruenberg–Kegel graph of a finite group

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Allover this talk by "group" we mean "a finite group" and by "graph" we mean "an undirected graph without loops and multiple edges".

Let G be a group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the spectrum of G, i.e., the set of all element orders of G. The set $\omega(G)$ defines the Gruenberg–Kegel graph (or the prime graph) $\Gamma(G)$ of G; in this graph, the vertex set is $\pi(G)$ and different vertices p and q are adjacent if and only if $pq \in \omega(G)$.

We say that a graph Γ with $|\pi(G)|$ vertices is realizable as the Gruenberg–Kegel graph of a group G if there exists a labeling the vertices of Γ by different primes from $\pi(G)$ such that the labeled graph is equal to $\Gamma(G)$. The following problem arises.

Problem. Let Γ be a graph. Is Γ realizable as the Gruenberg–Kegel graph of a finite group?

Of course, in general, Problem has a negative solution. For example, the graph consisting of five pairwise non-adjacent vertices (5-coclique) is not realizable as the Gruenberg–Kegel graph of a group. In [1] Problem was solved for graphs with at most 5 vertices. In [2] Problem was solved for complete bipartite graphs.

In this talk, we discuss (in progress) a solution of Problem for graphs with 6 vertices. There are 156 graphs with 6 vertices. Problem was solved for 106 of them.

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Finite groups with no elements of order 6

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In [1–3], the finite simple groups with no elements of order 6 had been classified. In the given work, a sufficiently complete description of the structure of general finite group with this property is obtained. We prove the following two theorems.

Theorem 1. *Let G be a finite solvable group with no elements of order 6 and 6 divides $|G|$. Then one of the following conditions holds:*

(1) $G/O(G)$ is either a cyclic or a (generalized) quaternion 2-group, a Sylow 3-subgroup of $O(G)$ is abelian, and the 3-length of $O(G)$ equals to 1;

(2) $G/O_{3'}(G)$ is either a cyclic 3-group or a dihedral group of order $2|G|_3$, the nilpotence class of a Sylow 2-subgroup of $O_{3'}(G)$ is at most 2, and the 2-length of $O_{3'}(G)$ is at most 1.

Theorem 2. *Let G be a finite non-solvable group and 3 divides $|G|$. Then G contains no elements of order 6 if and only if the group $O^{\{2,3\}'}(G/O_{\{2,3\}'}(G))$ is isomorphic to one of the following groups: $L_2(2^n)$; $L_2(3^n)$; $PGL_2(3^n)$; $L_2(3^{2k}).2_3$; $L_2(q)$, where $q \equiv \pm 5 \ (mod\ 12))$; $L_3(2^n)$, where $(2^n-1)_3 \leq 3))$; $U_3(2^n)$, where either 3 divide (2^n-1) or $(2^n+1)_3 = 3$; an extension of a non-trivial elementary abelian 2-group E by $L_2(2^n)$, where E considered as a $GF(2^n)L_2(2^n)$-module is isomorphic to a direct sum of natural $GF(2^n)L_2(2^n)$-modules.*

Note that in Theorem 2 $L_2(3^{2k}).2_3$ is a notation of the group $L_2(3^{2k})\langle df_1\rangle$, where $PGL_2(3^{2k}) = L_2(3^{2k})\langle d\rangle$ and f_1 is the involutive field automorphism of $L_2(3^{2k})$.

Theorem 2 implies the following corollary.

Corollary. *If G is an almost simple finite group with no elements of order 6 then G is isomorphic to an extension of its socle, or $PGL_2(3^n)$, or $L_2(3^{2k}).2_3$ by a field automorphism group of order coprime to 6.*

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Symmetrical 2-extensions of the 2-dimensional grid

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Let Λ^d be a d-dimensional grid. A connected graph Γ is called a symmetrical q-extension of Λ^d, if there exist a graph Δ of order q, a vertex-transitive automorphism group G of Γ and an imprimitivity system σ of the group G on the set of vertices of Γ, such that there is some isomorphism φ of the factor-graph Γ/σ on the grid Λ^d and the blocks of σ generates in Γ subgraphs isomprphic to Δ.

The investigation of symmetrical q-extensions of d-dimensional grids Λ^d is of interest both for group theory and graph theory. For small $d \geq 1$ and $q > 1$ (especially for $q = 2$), symmetrical q-extensions of Λ^d are also of interest for molecular crystallography and some phisycal theories. V. I. Trofimov proved in [1] that there are only finitely many symmetrical 2-extensions of Λ^d for any positive integer d. The aim of our work is to find all, up to isomorphism, symmetrical 2-extensions of Λ^2 (we show that there are 152 such extensions). Our work consists of two parts. In the first part of the work [2], we got all, up to isomorphism, symmetrical 2-extensions of Λ^2 such that only the trivial automorphism fixes all blocks of σ (78 extensions). In the second part of the work [3], we get all symmetrical 2-extensions of Λ^2 such that some non-trivial automorphism fixes all blocks of σ (75 extensions). One extension belongs both to the set of 78 extensions and to the set of 75 extensions mentioned above.

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On the question by Cameron about primitive permutation groups with stabilizer of two points that is normal in the stabilizer of one of them

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P. Cameron formulated the following question (see [1], [2, question 9.69]). Assume that G is a primitive permutation group on a finite set X, $x \in X$ and G_x acts regularly on the G_x-orbit $G_x(y)$ containing y for any $y \in X \setminus \{x\}$ (i.e. G_x induces on $G_x(y)$ a regular permutation group). Is it true that this action is faithful? (Note, that in some particular cases this question was treated earlier, see [3–5]).

In this talk, we report our recent results on the question.

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On chief factors of parabolic maximal subgroups of the group $B_l(2^n)$

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In previous papers [1]– [5], the author obtained a refined description of chief factors of parabolic maximal subgroups involved in the unipotent radical for all finite simple groups of Lie type (normal and twisted), except for special classical groups (defined below). We continue the study in this direction and consider the classical group $B_l(2^n)$.

Assume that G is a group of Lie type over a field of characteristic p and $P = UL$ is a parabolic maximal subgroup in G, where U is the unipotent radical and L is a Levi complement of P. We will say that G is *special* if $p = 2$ for groups of type B_l, C_l, and F_4 and $p \leqslant 3$ for groups of type G_2. It follows from the results of [6] that, for nonspecial groups G, factors of the lower central series of the group U are chief factors of the group P. In the exceptional cases, the commutator relations influencing the structure of unipotent subgroups behave in special way and require special consideration.

In present talk, the author refines the description of chief factors of each parabolic maximal subgroup of the finite simple group $B_l(2^n)$ involved in its unipotent radical.

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Unification of graph products and compatibility with switching

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We define the type of graph products, which enable us to treat many graph products in a unified manner. These unified graph products are shown to be compatible with Godsil–McKay switching. Furthermore, by this compatibility, we show that the Doob graphs can also be obtained from the Hamming graphs by switching.

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On resolvability at a point of topological spaces

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The notion of the resolvability at a point was introduced by E.G. Pytkeev [1]. A topological space X is called *resolvable at a point* $x \in X$ (*k-resolvable at a point* $x \in X$) if X contains k disjoint subsets A_t such that $x \in [A_t] \setminus \{x\}$.

S.P.Ponomarev [2] introduced a definition of local resolvability of a topological space X at a point $x \in X$. A topological space X is said to be local resolvable at a point $x \in X$ if each open neighborhood of x contains a nonempty open subset which is resolvable.

We will discuss the relationship between these definitions and the property of resolvability at a point for classical generalizations of compact spaces.

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Perfect colorings of the infinite n-multipath graph

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Let $G = (V, E)$ be a simple graph. A vertex partition $(V_1, V_2, \dots V_k)$ of the graph G is called a *perfect coloring* (equitable partition, partition design), if for every $i, j \in \{1, 2, \dots k\}$ there is a number m_{ij}, such that every vertex from V_i has exactly m_{ij} neighbors from V_j. The matrix $M = (m_{ij})$ is called the *parameter matrix* of the coloring.

A *co-normal product* of graphs is an operation, that takes two graphs G and H and produces a graph $G * H$ with the following properties: the vertex set of $G * H$ is the cartesian product $V(G) \times V(H)$, the vertices (u_1, u_2) and (v_1, v_2) are connected by an edge if and only if $\{u_1, v_1\} \in E(G)$ or $\{u_2, v_2\} \in E(H)$.

Consider an infinite graph C_∞, whose set of vertices is the set of integers, and two vertices are adjacent, if they are on the distance 1. Let n be a positive integer. An infinite n-multipath graph is a co-normal product $C_\infty * \overline{K_n}$ of the graph C_∞ and the empty graph $\overline{K_n}$. Perfect colorings of the graph $C_\infty * \overline{K_n}$ combine properties of perfect colorings of both multipliers. Let us consider them closely.

Any coloring of the graph C_∞ evidently is periodic. We note the period of a coloring of this graph by the string in brackets, whose length is the number of elements in the period. Colorings of the graph C_∞ with periods $S_{11}(k) = [1\ 2\ 3\ \dots\ (k-1)\ k\ (k-1)\ \dots\ 3\ 2]$, $S_{12}(k) = [1\ 2\ 3\ \dots\ (k-1)\ k\ k\ (k-1)\ \dots\ 3\ 2]$ and $S_{22}(k) = [1\ 2\ 3\ \dots\ (k-1)\ k\ k\ (k-1)\ \dots\ 3\ 2\ 1]$ are called *mirror colorings*, and with periods $S(k) = [1\ 2\ 3\ \dots\ k]$ — *cyclic colorings*.

The complete description of perfect colorings of this graph is well known.

Lemma. *Perfect colorings of the graph C_∞ are exhausted by following infinite series: three series of the mirror colorings and one series of the cyclic colorings.*

Note, that every coloring of the empty graph with n vertices is perfect by definition. It is clear also, that every coloring of the graph $C_\infty * \overline{K_n}$ is periodic.

Let us construct a coloring of the infinite n-multipath graph. We take perfect coloring of the graph C_∞. Then we assign a vertex partition of the empty graph to every color in it so as different colors in initial coloring correspond to colorings of $\overline{K_n}$ with disjoint sets of colors. We put copies of graph $\overline{K_n}$ colored in that way into suitable place. This structure on the graph $C_\infty * \overline{K_n}$ is called *disjunctive coloring*.

Let us describe another construction for this graph. We consider a coloring of the graph $C_\infty * \overline{K_n}$ with period of length 4, fix one period. For each color i we compose a vector of length 4, whose components are numbers of i-colored vertices in corresponding copies of $\overline{K_n}$. We call this vector the *i-characteristic vector*. If the sum of components of i-characteristic vector with even numbers is equal to the sum of components with odd numbers for every i, the coloring is called *nonstandard*.

The following theorem holds:

Theorem. *Disjunctive and nonstandard colorings of the graph $C_\infty * \overline{K_n}$ are perfect.*

We conjecture, that there are no other perfect colorings of the infinite n-multipath graph.

Vetex-symmetric semitriangular Higman graph with $\mu = 7$

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We consider undirected graphs without loops and multiple edges. For a vertex a of a graph Γ the subgraph $\Gamma_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighborhood of a in Γ. Edge-symmetric semitriangular Higman graphs were classified in [1]. We begin to investigate vertex-symmetric semitriangular Higman graphs. In this work semitriangular Higman graph with $\mu = 7$ were investigated.

Theorem 1. *Let Γ be a vertex-symmetric strongly regular graph with parameters $(1275, 98, 13, 7)$. If the group $G = \mathrm{Aut}(\Gamma)$ is nonsolvable, $\bar{T} = socle(G/S(G))$, and 17 divides $|\bar{T}|$ then one of the following statement holds:*

(1) *$\bar{T} \cong Sp_8(2)$, $V = S(G)$ is an elementary abelian 5-group, $|V : V_a| = 5$, and $Sp_8(2)$ acts irreducible on V;*

(2) *$\bar{T} \cong L_4(4)$ and one of the following statements holds:*

(*i*) *$\bar{T}_a$ is an extension of E_{64} by $SL_3(4)$, $V = S(G)$ is an elementary abelian 5-group, $|V : V_a| = 5$, and $L_4(4)$ acts irreducible on V,*

(*ii*) *$\bar{T}_a$ is an extension of E_{64} by $GL_3(4)$, $S(G) = R \times V$, where R is an elementary abelian 3-group and V is an elementary abelian 5-group, $|R : R_a| = 3$ and $L_4(4)$ acts irreducible on R, $|V : V_a| = 5$ and $L_4(4)$ acts irreducible on V;*

(3) *$\bar{T} \cong Sp_4(4)$ and one of the following statements holds:*

(*i*) *$\bar{T}_a$ is an extension of E_{64} by A_5, $V = S(G)$ is an elementary abelian 5-group, $|V : V_a| = 5$, and $Sp_4(4)$ acts irreducible on V,*

(*ii*) *$\bar{T}_a$ is an extension of E_{64} by $Z_3 \times A_5$, $S(G) = R \times V$, where R is an elementary abelian 3-group and V is an elementary abelian 5-group, $|R : R_a| = 3$ and $Sp_4(4)$ acts irreducible on R, $|V : V_a| = 5$ and $Sp_4(4)$ acts irreducible on V;*

(4) *$\bar{T} \cong L_2(16)$ and one of the following statements holds:*

(*i*) *$\bar{T}_a$ is a group of order 16, $V = S(G)$ is an elementary abelian 5-group, $|V : V_a| = 5$, and $L_2(16)$ acts irreducible on V,*

(*ii*) *$\bar{T}_a$ is an extension of E_{16} by Z_3, $S(G) = R \times V$, where R is an elementary abelian 3-group and V is an elementary abelian 5-group, $|R : R_a| = 3$ and $L_2(16)$ acts irreducible on R, $|V : V_a| = 5$ and $L_2(16)$ acts irreducible on V,*

(*iii*) *$\bar{T}_a$ is an extension of E_{16} by Z_5, $S(G)$ is 5-group, and $|S(G) : S(G)_a| = 25$,*

(*iv*) *$\bar{T}_a$ is an extension of E_{16} by Z_{15}, if $R \in Sylow_3(S(G))$ then R is an elementary abelian 3-group, $|R : R_a| = 3$ and $L_2(16)$ acts irreducible on R, if $R \in Sylow_5(S(G))$ then $|R : R_a| = 25$.*

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Automorphisms of graph with intersection array $\{69, 56, 10; 1, 14, 60\}$

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We consider undirected graphs without loops and multiple edges. For a vertex a of a graph Γ the subgraph $\Gamma_i(a) = \{b \mid d(a, b) = i\}$ is called i-neighborhood of a in Γ. Let $[a] = \Gamma_1(a)$.

Degree of a vertex a in Γ is the number of vertices in $[a]$. A graph Γ is called regular of degree k if degree of any vertex is equal to k. A graph Γ is called amply regular with parameters (v, k, λ, μ) if Γ is regular of degree k on v vertices, $|[u] \cap [w]|$ is equal to λ if u adjacent to w and is equal to μ if $d(u, w) = 2$. Amply regular graph of diameter 2 is called strongly regular.

Let Γ be a distance-regular graph of diameter 3 with eigenvalues $\theta_0 > \theta_1 > \theta_2 > \theta_3$. If $\theta_2 = -1$, then by [1, proposition 4.2.17] graph Γ_3 is strongly regular. If also Γ_2 is strongly regular graph without triangles and $v < 800$, then Γ has intersection array $\{69, 56, 10; 1, 14, 60\}$. Moreover Γ_3 has parameters (392,46,0,6) and $\bar{\Gamma}_2$ has parameters (392,115,18,40).

In this work automorphisms of distance-regular graph with the intersection array $\{69, 56, 10; 1, 14, 60\}$ were founded.

Theorem 1. *Let Γ be a distance-regular graph with the intersection array $\{69, 56, 10; 1, 14, 60\}$, $G = \mathrm{Aut}(\Gamma)$, g is an element of G of prime order p, and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 7, 23\}$ and one of the following holds:*

(1) *Ω is the empty graph and either $p = 7$, $\alpha_3(g) = 98s$, $\alpha_2(g) = 198t$, or $p = 2$, $\alpha_3(g) = 28s$, $\alpha_2(g) = 56t$;*
(2) *$|\Omega| = 1$, $p = 23$, $\alpha_1(g) = 69$, $\alpha_2(g) = 276$, and $\alpha_3(g) = 46$;*
(3) *$|\Omega| = 21s + 14$, $p = 3$, $s \in \{0, 1, 2\}$, $\alpha_3(g) = 0$, and $\alpha_2(g) = 84t$.*

Corollary. *Let Γ be a distance-regular graph with the intersection array $\{69, 56, 10; 1, 14, 60\}$. If $G = \mathrm{Aut}(\Gamma)$ acts transitively on the vertex set of Γ then either $|G| = 8 \cdot 49$ and Γ is a Cayley graph, or either $G = Z(G) \times L$, $Z(G) \cong Z_7$, $L \cong L_2(7), L_2(8)$, and $L_a \in Sylow_3(L)$, or G contains a subgroup H of index 2, $H \cong Z_7 \times L_2(7)$, $|H_a| = 6$, and $G/S(G) \cong PGL_2(7)$.*

In the proof of Theorem 1 we used following result.

Theorem 2. *Let Γ be a strongly regular graph with parameters $(392, 46, 0, 6)$, $G = \mathrm{Aut}(\Gamma)$, g is an element of G of prime order p, and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 23\}$ and one of the following holds:*

(1) *Ω is the empty graph, either $p = 7$ and $\alpha_1(g) = 46$, or $p = 2$ and $\alpha_1(g) = 28t$;*
(2) *Ω is an n-clique, either $n = 1$, $p = 23$, and $\alpha_1(g) = 46$, or $n = 2$, $p = 5$, and $\alpha_1(g) = 70l - 20$;*
(3) *Ω is an m-coclique, $4 \le m \le 56$, $p = 2$, and $\alpha_1(g) = 28l - 10m$;*
(4) *Ω is a union of l isolated edges, $l \in \{7, 28\}$, $p = 3$, and $\alpha_1(g) = 0$;*
(5) *Ω contains a geodesic 2-way and $p \le 5$.*

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Automorphisms of distance-regular graph with intersection array $\{176, 135, 32, 1; 1, 16, 135, 176\}$

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A distance-regular graph Γ with intersection array $\{176, 135, 32, 1; 1, 16, 135, 176\}$ is $AT4(8,4,3)$-graph [1]. The antipodal quotient $\bar{\Gamma}$ has parameters $(672, 176, 40, 48)$. In this work automorphisms of a distance-regular graph Γ with intersection array $\{176, 135, 32, 1; 1, 16, 135, 176\}$ and of its antipodal quotient $\bar{\Gamma}$ were investigated.

Theorem 1. *Let Γ be a strongly regular graph with parameters $(672, 176, 40, 48)$, $G = \mathrm{Aut}(\Gamma)$, g is an element of G of prime order p, and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$ and one of the following holds:*
(1) *Ω is the empty graph, $p \in \{2, 3, 7\}$;*
(2) *Ω is an m-coclique, $p = 11$ and $m = 1$ or $p = 2$ and m is even;*
(3) *Ω is an n-clique, $p = 3$, and $n = 3t$;*
(4) *Ω contains geodesic 2-way and one of the following holds:*
(*i*) *$p = 7$, $|\Omega| = 7s$, and $s \le 27$;*
(*ii*) *$p = 5$, $|\Omega| = 5s + 2$, and $s \le 38$;*
(*iii*) *$p = 3$, $|\Omega| = 3s$, and $s \le 64$;*
(*iv*) *$p = 2$, $|\Omega| = 2s$, and $s \le 96$.*

Corollary 1. *Let Γ be a vertex-symmetric strongly regular graph with parameters $(672, 176, 40, 48)$. If $G = \mathrm{Aut}(\Gamma)$ contains an element of order 11, $S(G) = 1$, and $T = socle(G)$. Then $T \cong U_6(2).Z_6$, $T_a \cong U_5(2).Z_6$, and Γ is rank 3-graph.*

Theorem 2. *Let Γ be a distance-regular graph with the intersection array $\{176, 135, 32, 1; 1, 16, 135, 176\}$, $G = \mathrm{Aut}(\Gamma)$, g is an element of G of prime order p, and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 7, 23\}$ and one of the following holds:*
(1) *g induces a trivial automorphism of antipodal quotient $\bar{\Gamma}$, $p = 3$, and $\alpha_4(g) = 2016$;*
(2) *Ω is the empty graph, $\alpha_4(g) = 0$, and $p \in \{2, 3, 7\}$;*
(3) *$\bar{\Omega}$ is an m-coclique, either $p = 11$ and $m = 1$ or $p = 2$ and m is even;*
(4) *$\bar{\Omega}$ is an n-clique, $p = 3$, and $n \in \{6, 12\}$;*
(5) *Ω contains geodesic 2-way and either one of the following holds:*
(*i*) *$p = 7$, $\alpha_4(g) = 0$, $|\Omega| = 21s$, where $s \le 27$;*
(*ii*) *$p = 5$, $\alpha_4(g) = 0$, and $|\Omega| = 15s + 6$;*
(*iii*) *$p = 3$, $|\Omega| + \alpha_4(g) = 9s$, where $s \le 64$;*
(*iv*) *$p = 2$, $|\Omega| + \alpha_4(g) = 6s$, where $s \le 96$.*

Corollary 2. *Let Γ be a vertex-symmetric distance-regular graph with the intersection array $\{176, 135, 32, 1; 1, 16, 135, 176\}$. If $G = \mathrm{Aut}(\Gamma)$ contains an element of order 11 and $S(G)$ fixes every antipodal class then the full preimage of $(G/S(G))'$ is an extension of a group of order 3 either by M_{22} or by $U_6(2)$.*

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Recognizing small finite simple groups by element orders in the class of all groups

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Spectrum $\omega(G)$ of a periodic group G is the set of its element orders. Consider the following question: given that H is a finite simple group and $\omega(G) = \omega(H)$, what can we say about G, for example, when $G \simeq H$? There is a lot of research and progress if G is assumed to be finite apriory, and there are only few results in general case, when it was possible to resolve corresponding Burnside problem. Precisely, the following groups are known to be recognizible by their element orders in the class of all groups: $L_2(2^m)$ [1], $L_2(7) \simeq L_3(2)$ [2], $L_3(4)$ [3].

In the talk we discuss current progress in the case $\omega(G) = \omega(A_7) = \{1, 2, 3, 4, 5, 6, 7\}$.

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Revision of the classification of maximal subgroups of odd index in finite simple groups

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Liebeck and Saxl [1] and, independently, Kantor [2] proposed a classification of finite primitive permutation groups of odd degree. It was one of the greatest results in the theory of finite permutation groups.

Both papers [1] and [2] contain lists of subgroups of finite simple groups that can turn out to be maximal subgroups of odd index. However, in the cases of alternating groups and of classical groups over fields of odd characteristics, neither in [1] nor in [2] it was described which of the specified subgroups are precisely maximal subgroups of odd index. Thus, the problem of the complete classification of maximal subgroups of odd index in finite simple groups remained open. The classification was finished by the author in [3, 4].

In [3] we used results obtained by Kleidman in [5]. However there is a number of mistakes and inaccuracies in [5]. These mistakes and inaccuracies were corrected in [6].

In this talk we discuss a revision of the classification of maximal subgroups of odd index in finite simple classical groups obtained in [3].

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A pronormality criterion for subgroups of odd indices in a family of finite groups

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Join work with Wenbin Guo and Danila O. Revin

A subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in G$.

In [1] Vdovin and Revin conjectured that all subgroups of odd indices are pronormal in all finite simple groups. The conjecture was verified for many families of finite simple groups in [2]. However in [3] it was proved that the conjecture fails. The problem of classification of finite simple groups in which all subgroups of odd indices are pronormal naturally arises. In [4] this problem was solved for almost all finite simple symplectic groups. We continue to investigate the problem which is still open for some finite simple groups of Lie type over fields of odd characteristics.

In this talk we discuss a pronormality criterion for subgroups of odd indices in finite groups of the type $\prod_{i=1}^{t}(A \wr Sym(n_i))$, where A is an abelian group and all the wreath products are natural permutation. This result is an useful tool in the research of the problem of classification of finite simple groups in which all subgroups of odd indices are pronormal.

Let $\mathbb{M}$ be the set of all sequences $(x_0, x_1, ..., x_n, ...)$ such that $x_i \in \{0, 1\}$ for all i and the number of nonzero components is finite. Let us introduce on $\mathbb{M}$ the natural order $\succeq$ as follows: $1 \succeq 0$ and, for $u = (u_0, u_1, ..., u_n, ...)$ and $v = (v_0, v_1, ..., v_n, ...)$ from $\mathbb{M}$, the relation $u \succeq v$ holds if and only if $u_i \succeq v_i$ for all i. We denote by ψ the function that takes each nonnegative integer s to the sequence $(s_0, s_1, ..., s_k, ...)$ from $\mathbb{M}$ such that $s_k s_{k-1} ... s_0$ is the binary notation for the number s and $s_n = 0$ for all $n > k$.

We prove the following theorem.

Theorem. *Let A be a finite abelian group and $G = \prod_{i=1}^{t}(A \wr Sym(n_i))$, where all the wreath products are natural permutation. Then all subgroups of odd indices are pronormal in G if and only if for any positive integer m the following condition holds: if $\psi(n_i) \succeq \psi(m)$ for some i then h.c.f.$(|A|, m)$ is a power of 2.*

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Program realization of the classification of maximal subgroups of odd index in finite simple classical groups

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A classification of maximal subgroups of odd index in finite simple groups was proposed by Liebeck and Saxl [1] and, independently, Kantor [2]. The classification was completed by the thirst author in [3, 4].

The ariphmetical criterium obtained in [3] is useful, but hard-in-calculating in some cases. In this talk we discuss a program realization of the classification of maximal subgroups of odd index in finite simple classical groups.

The project architecture is a dynamic library that implements an interface and algorithms of calculating of maximal subgroups of odd indices for finite simple classical groups. The library exports functions convenient for working with the criterium obtained in [3]. The dynamic library is cross platform and could be assembled under different operating systems which is very convenient. For calculations, the GMP library was used. To test the work of algorithms, modular tests were written with using the Microsoft unit testing platform for C++. Based on input data tests check all the boundary cases for all exported algorithms.

For the convenience of working under the windows operating system, an interface for Windows Presentation Foundation was implemented. The interface validates the input data and helps the user to use the library.

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Cameron-Liebler line classes in $PG(3,5)$

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This is joint work with Alexander Gavrilyuk

Let $\mathrm{PG}(n,q)$ denote the n-dimensional projective space over the finite field $\mathbb{F}_q$. A set $\mathcal{L}$ of lines of $\mathrm{PG}(n,q)$ is a *Cameron-Liebler* line class [4] if there exists $x \in \mathbb{Q}$, $x \geq 0$, such that every line ℓ of $\mathrm{PG}(n,q)$ intersects $x(q+1)$ lines of $\mathcal{L}$ if $\ell \notin \mathcal{L}$, and $x(q+1)+q^{n-1}+\ldots+q^2-1$ lines of $\mathcal{L}\setminus\{\ell\}$ if $\ell \in \mathcal{L}$.

Any collineation group $G \leq P\Gamma L(n+1,q)$ has at least as many orbits on the lines of $\mathrm{PG}(n,q)$ as on the points. Cameron and Liebler [3] in their attempt to classify those collineation groups of $\mathrm{PG}(n,q)$, $n \geq 3$, that have equally many orbits on lines and on points observed that a line orbit of such a group should be a Cameron-Liebler line class. Furthermore, they conjectured that such a line class $\mathcal{L}$ or its complement $\mathrm{PG}(n,q)\setminus\mathcal{L}$ is an empty set of lines, or the set of lines in a hyperplane or through a point, or the union of lines in a hyperplane or through a point if the point is not in the hyperplane.

The first counterexample to the conjecture was constructed by Drudge [5] in $\mathrm{PG}(3,3)$ and generalized in [2] in $\mathrm{PG}(3,q)$ for all odd q. Many more non-trivial examples of Cameron-Liebler line classes in $\mathrm{PG}(3,q)$ were found in [1, 6–8, 10, 11]. Despite that, for $n>3$, the conjecture in $\mathrm{PG}(n,q)$ remains open.

The intersection of a Cameron-Liebler line class in $\mathrm{PG}(n,q)$, $n>3$, with a 3-dimensional subspace of $\mathrm{PG}(n,q)$ is a Cameron-Liebler line class in $\mathrm{PG}(3,q)$. Thus, for a given q, having obtained a list of all Cameron-Liebler line classes in $\mathrm{PG}(3,q)$, one can try to confirm the conjecture in $\mathrm{PG}(n,q)$ for $n>3$. For $q \in \{2,3,4\}$ and $n>3$, the conjecture in $\mathrm{PG}(n,q)$ was confirmed in [3, 4, 9]. In this work we determined all Cameron-Liebler line classes in $\mathrm{PG}(3,5)$, which would provide a basis to confirm the conjecture in $\mathrm{PG}(n,5)$, $n>3$.

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On piecewise continuous mappings of paracompact spaces

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We shall discuss a generalization of the following theorem due to J.E. Jayne and C.A. Rogers.

Theorem 1. [1] *If X is an absolute Souslin-$\mathcal{F}$ set and Y is a metric space, then $f\colon X \to Y$ is $\mathbf{\Delta}^0_2$-measurable if and only if it is piecewise continuous.*

Kačena, Motto Ros, and Semmes [2] showed that Theorem 1 holds for a regular space Y.

Recall that a metrizable space X is said to be *an absolute Souslin-$\mathcal{F}$* set if X is a result of the $\mathcal{A}$-operation applied to a system of closed subsets of $\tilde{X}$, where $\tilde{X}$ is the completion of X under its compactible metric. A space X is called *perfectly paracompact* if X is paracompact and each closed subset of X is of type G_δ in X.

A mapping $f\colon X \to Y$ is said to be

- $\mathbf{\Delta}^0_2$*-measurable* if $f^{-1}(U) \in \mathbf{\Delta}^0_2(X)$ for every open set $U \subset Y$,
- *piecewise continuous* if X can be covered by a sequence $X_0, X_1, \ldots$ of closed sets such that the restriction $f|X_n$ is continuous for every $n \in \omega$.

We show how Theorem 1 can be extended to a perfectly paracompact space X satisfying the first axiom of countability.

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Reductive homogeneous spaces and connections on them

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When a homogeneous space admits an invariant affine connection? If there exists at least one invariant connection then the space is isotropy-faithful, but the isotropy-faithfulness is not sufficient for the space in order to have invariant connections. If a homogeneous space is reductive, then the space admits an invariant connection. The purpose of the work is the classification of three-dimensional reductive homogeneous spaces and invariant affine connections on them.

Let $(\overline{G}, M)$ be a three-dimensional homogeneous space, where $\overline{G}$ is a Lie group on the manifold M. We fix an arbitrary point $o \in M$ and denote by $G = \overline{G}_o$ the stationary subgroup of o. The problem of classification of homogeneous spaces $(\overline{G}, M)$ is equivalent to the classification (up to equivalence) of pairs of Lie groups $(\overline{G}, G)$. Since we are interested in only the local equivalence problem, we can assume without loss of generality that both $\overline{G}$ and G are connected. Then we can correspond the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ of Lie algebras to $(\overline{G}, M)$, where $\bar{\mathfrak{g}}$ is the Lie algebra of $\overline{G}$ and $\mathfrak{g}$ is the subalgebra of $\bar{\mathfrak{g}}$ corresponding to the subgroup G. This pair uniquely determines the local structure of $(\overline{G}, M)$, two homogeneous spaces are locally isomorphic if and only if the corresponding pairs of Lie algebras are equivalent. An *isotropic* $\mathfrak{g}$*-module* $\mathfrak{m}$ is the $\mathfrak{g}$-module $\bar{\mathfrak{g}}/\mathfrak{g}$ such that $x.(y+\mathfrak{g}) = [x, y]+\mathfrak{g}$. The corresponding representation $\lambda\colon \mathfrak{g} \to \mathfrak{gl}(\mathfrak{m})$ is called an *isotropic representation* of $(\bar{\mathfrak{g}}, \mathfrak{g})$. The pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is said to be *isotropy-faithful* if its isotropic representation is injective. Invariant affine connections on $(\overline{G}, M)$ are in one-to-one correspondence [2] with linear mappings $\Lambda\colon \bar{\mathfrak{g}} \to \mathfrak{gl}(\mathfrak{m})$ such that $\Lambda|_{\mathfrak{g}} = \lambda$ and Λ is $\mathfrak{g}$-invariant. We call this mappings *(invariant) affine connections* on the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$. If there exists at least one invariant connection on $(\bar{\mathfrak{g}}, \mathfrak{g})$ then this pair is isotropy-faithful [3]. We say that a homogeneous space $\overline{G}/G$ is *reductive* if the Lie algebra $\bar{\mathfrak{g}}$ may be decomposed into a vector space direct sum of the Lie algebra $\mathfrak{g}$ and an $\mathrm{ad}(G)$-invariant subspace $\mathfrak{m}$, that is, if $\bar{\mathfrak{g}} = \mathfrak{g} + \mathfrak{m}$, $\mathfrak{g} \cap \mathfrak{m} = 0$ and $\mathrm{ad}(G)\mathfrak{m} \subset \mathfrak{m}$. Last condition implies $[\mathfrak{g}, \mathfrak{m}] \subset \mathfrak{m}$ and, conversely, if G is connected. If a homogeneous space is reductive, then the space always admits an invariant connection. In any of the following cases a homogeneous space $\overline{G}/G$ is reductive [3]: G is compact; G is connected and $\mathfrak{g}$ is reductive in $\bar{\mathfrak{g}}$; G is a discrete subgroup of $\overline{G}$. The curvature and torsion tensors of the invariant affine connection Λ are given by the following formulas: $R\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{gl}(\mathfrak{m})$, $(x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto [\Lambda(x_1), \Lambda(x_2)] - \Lambda([x_1, x_2])$; $T\colon \mathfrak{m} \wedge \mathfrak{m} \to \mathfrak{m}$, $(x_1+\mathfrak{g}) \wedge (x_2+\mathfrak{g}) \mapsto \Lambda(x_1)(x_2+\mathfrak{g}) - \Lambda(x_2)(x_1+\mathfrak{g}) - [x_1, x_2]_{\mathfrak{m}}$.

We divide the solution of the problem of classification all three-dimensional reductive pairs $(\bar{\mathfrak{g}}, \mathfrak{g})$ into the following parts. We classify (up to isomorphism) faithful three-dimensional $\mathfrak{g}$-modules U. This is equivalent to classifying all subalgebras of $\mathfrak{gl}(3, \mathbb{R})$ viewed up to conjugation. For each obtained $\mathfrak{g}$-module U we classify (up to equivalence) all pairs $(\bar{\mathfrak{g}}, \mathfrak{g})$ such that the $\mathfrak{g}$-modules $\bar{\mathfrak{g}}/\mathfrak{g}$ and U are isomorphic. All there pairs are described in [1]. From all isotropy-faithful pairs we choose reductive pairs.

We describe all local three-dimensional reductive homogeneous spaces, it is equivalent to the description of effective pairs of Lie algebras, and all invariant affine connections on the spaces together with their curvature, torsion tensors and holonomy algebras. The results of work can be used in research work of the differential geometry, differential equations, topology, in the theory of representations, in the theoretical physics.

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Graph complexity and tetrahedral complexity of compact 3-manifolds

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The graph complexity $c_g(M)$ of a compact 3-manifold M is the minimum order among all 4-colored graphs representing the manifold, while the tetrahedral complexity $c_{tet}(M)$ is the minimum number of tetrahedra in a (pseudo) triangulation of M. By construction $c_{tet}(M) \leq c_g(M)$ and in the closed case the inequality is always strict, but in the case of hyperbolic manifolds with toric boundary the two invariants often coincide. In this talk we describe an infinite family of 3-manifolds of this type and compute the value of their complexity. Moreover, we present the census of compact orientable prime 3-manifolds with toric boundary, up to graph complexity 14.

Reidemeister spectrum of special and general linear groups over some fields contains 1

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Let G be a group and φ be an endomorphism of G. Elements x, y from G are called *φ-conjugated* if there exists an element $z \in G$ such that $x = zy\varphi(z)^{-1}$. The relation of φ-conjugation is an equivalence relation and it divides G into *φ-conjugacy classes*. The number $R(\varphi)$ of these classes is called *the Reidemeister number of the endomorphism φ*. The Reidemeister number is either a positive integer or infinity and we do not distinguish different infinite cardinal numbers denoting all of them by the symbol ∞. The set $\{R(\varphi) \mid \varphi \text{ is an automorphism of } G\}$ is called *the Reidemeister spectrum of G* and is denoted by $Spec_R(G)$. If $Spec_R(G) = \{\infty\}$, then G is said to *possess the R_∞-property*.

The problem of determining groups which possess the R_∞-property was formulated by A. Fel'shtyn and R. Hill [1]. Some aspects of the R_∞-property, namely, relation with nonabelian cohomology, relation with isogredience classes and relation with representation theory can be found in [3].

The author studied conditions which imply the R_∞-property for different linear groups. In particular, it was proved that special linear group $\mathrm{SL}_n(\mathbb{F})$ and general linear group $\mathrm{GL}_n(\mathbb{F})$ over a field $\mathbb{F}$ of zero characteristic possess the R_∞-property if $\mathbb{F}$ is an algebraically closed field of zero characteristic which has finite transcendence degree over $\mathbb{Q}$ [2, 5], or if the auromorphism group of $\mathbb{F}$ is periodic [2, 4]. Some fields of zero characteristic with infinite transcendence degree over $\mathbb{Q}$ have periodic groups of automorphisms, however, if $\mathbb{F}$ is an algebraically closed field of zero characteristic which has infinite transcendence degree over $\mathbb{Q}$, then it always has an automorphism of infinite order. So, the case of linear groups over algebraically closed fields of zero characteristic with infinite transcendence over $\mathbb{Q}$ is absolutely not studied.

In the talk we are going to discuss twisted conjugacy classes and the Reidemeister spectrum for special linear group $\mathrm{SL}_n(\mathbb{F})$ and general linear group $\mathrm{GL}_n(\mathbb{F})$ over an algebraically closed field $\mathbb{F}$ of zero characteristic which has infinite transcendence degree over the field of rational numbers $\mathbb{Q}$. One of the main results which we are going to introduce is the following theorem.

Theorem. *Let $\mathbb{F}$ be an algebraically closed field of zero characteristic with infinite transcendence degree over $\mathbb{Q}$. Then $Spec_R(\mathrm{SL}_n(\mathbb{F}))$ and $Spec_R(\mathrm{GL}_n(\mathbb{F}))$ contain* 1.

Also during the talk we will discuss some related results and formulate several open problems.

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On algebraic structure of a space of $\mathbb{R}^\tau$-valued continuous functions with the set-open topology

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Let X be a Tychonoff space and τ be a cardinal number. We shall denote by $C(X,\mathbb{R}^\tau)$ the set of all continuous mappings of the space X to the space $\mathbb{R}^\tau$. We say that a subset B of a space X is C_τ-compact in X if for every continuous function $f : X \mapsto \mathbb{R}^\tau$, $f(B)$ is a compact subset of $\mathbb{R}^\tau$ ([6], [7]). A family λ of C_τ-compact subsets of X is said to be closed under (hereditary with respect to) C_τ-compact subsets if it satisfies the following condition: whenever $A \in \lambda$ and B is a C_τ-compact (in X) subset of A, then $B \in \lambda$ also.

We use the following notations for various topological spaces on the set $C(X,\mathbb{R}^\tau)$: $C_{\hat{\mathcal{U}}|\lambda}(X,\mathbb{R}^\tau)$ for the topology induced by the uniformity $\hat{\mathcal{U}}|\lambda$, $C_\lambda(X,\mathbb{R}^\tau)$ for the λ-open topology. The element of the standard subbase of the λ-open topology: $[F,U] = \{f \in C(X,\mathbb{R}^\tau) : \ f(F) \subseteq U\}$ where $F \in \lambda$ and U is a open subset of $\mathbb{R}^\tau$. Given a family λ of non-empty subsets of X, let $\lambda(C_\tau) = \{A \in \lambda :$ for every C_τ-compact subset B of the space X with $B \subset A$, the set $[B,U]$ is open in $C_\lambda(X,\mathbb{R}^\tau)$ for any open set U of the space $\mathbb{R}^\tau\}$. Let λ_m denote the maximal with respect to inclusion family, provided that $C_{\lambda_m}(X,\mathbb{R}^\tau) = C_\lambda(X,\mathbb{R}^\tau)$.

In this paper, we look at the properties of the family λ which imply that the space $C(X,\mathbb{R}^\tau)$ with the λ-open topology is a semitopological group (topological vector space, topological group and other algebraic structures) under the usual operations of addition and multiplication (and multiplication by scalars). The main result of the study is the following

Theorem 1 For a Tychonoff space X and a cardinal number τ, the following statements are equivalent.
(1) $C_\lambda(X,\mathbb{R}^\tau)$ is a semitopological group.
(2) $C_\lambda(X,\mathbb{R}^\tau)$ is a paratopological group.
(3) $C_\lambda(X,\mathbb{R}^\tau)$ is a topological group.
(4) $C_\lambda(X,\mathbb{R}^\tau)$ is a topological vector space.
(5) $C_\lambda(X,\mathbb{R}^\tau)$ is a locally convex topological vector space.
(6) $C_\lambda(X,\mathbb{R}^\tau)$ is a topological ring.
(7) $C_\lambda(X,\mathbb{R}^\tau)$ is a topological algebra.
(8) λ is a family of C_τ-compact sets and $\lambda = \lambda(C_\tau)$.
(9) λ_m is a family of C_τ-compact sets and it is hereditary with respect to C_τ-compact subsets.
(10) $C_\lambda(X,\mathbb{R}^\tau) = C_{\hat{\mathcal{U}}|\lambda}(X,\mathbb{R}^\tau)$.

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On integral Cayley graphs for A_n

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We consider the problem of finding integral Cayley graphs for alternating groups A_n for $n = 4, 5, 6, 7$.

The Cayley graph $\Gamma = Cay(G, S) = (V, E)$ on a group G with a generators set S is a graph with the vertex set $V = G$ and the set of edges $E = \{\{g, h\} : g, h \in G, g^{-1}h \in S\}$. The spectrum of a graph Γ is defined as the set of real eigenvalues of its adjacency matrix [1].

According to the definition introduced by F. Harary and A. J. Schwenk in 1974 in [2], the graph G is said to be integral if its spectrum consists of integers. In the same paper, they set the task of finding integral Cayley graphs. The focus of our study is the Cayley graphs on alternating groups A_n for different n.

Theorem. *The following Cayley graphs on alternating groups are integral:*

1) $\Gamma_1 = Cay(G, S)$, where $G = A_4$, $S = \{(123), (124)\}$ or $S = \{(123), (234)\}$ or $S = \{(123), (134)\}$ or $S = \{(123), (12)(34)\}$.

2) $\Gamma_2 = Cay(G, S)$, where $G = A_5$, $S = \{(123), (124), (125)\}$.

3) $\Gamma_3 = Cay(G, S)$, where $G = A_6$, $S = \{(123), (124), (125), (126)\}$.

4) $\Gamma_4 = Cay(G, S)$, where $G = A_7$, $S = \{(123), (124), (125), (126), (127)\}$.

Conjecture (D. V. Lytkina). *Let $\Gamma = Cay(G, S)$, where $G = A_n$, $S = \{(12i) | i = 3, \dots, n\}$, $n \geqslant 3$. Then Γ is integral.*

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Cycles in the 3-Big Pancake graphs

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This is joint work with S. Goryainov, E. Konstantinova and A. Pankratova

In 2016, J. Sawada and A. Williams [3] conjectured that the 3-Big Pancake graphs $P_n^{big_3}$, $n \geqslant 4$, are hamiltonians. The 3-Big Pancake graph is defined as a Cayley graph on the symmetric group Sym_n of permutations with the generating set $big_3 = \{r_n, r_{n-1}, r_{n-2}\}$ of three biggest prefix–reversals. By the definition, it is a cubic graph. Its connectivity was proved by D. W. Bass and I. H. Sudborough [1] in 2003 as one of Cayley graphs on the symmetric group generated by restricted prefix–reversals.

In this work we investigate a cycle structure of the 3-Big Pancake graphs $P_n^{big_3}$. A sequence of prefix–reversals $C_\ell = r_{i_0} \dots r_{i_{\ell-1}}$, where $n-2 \leqslant i_j \leqslant n$, and $i_j \neq i_{j+1}$ for any $0 \leqslant j \leqslant \ell - 1$, such that $\pi\, r_{i_0} \dots r_{i_{\ell-1}} = \pi$, where $\pi \in \mathrm{Sym}_n$, is called a form of a cycle C_ℓ of length ℓ. The canonical form C_ℓ of an l–cycle is called a form with a lexicographically maximal sequence of indices $i_0 \dots i_{\ell-1}$. We characterize cycles in the 3-Big Pancake graphs in terms of their canonical forms.

We prove the following results.

Theorem 1. *There are no cycles of length* $6, 7$ *or* 9 *in* $P_n^{big_3}$ *for* $n \geqslant 6$.

Theorem 2. *There are cycles of length* $6, 7, 8$ *or* 9 *given by eleven distinct canonical forms in* $P_n^{big_3}$ *for* $n = 4, 5$. *Each of vertices of* $P_n^{big_3}$, $n \geqslant 6$, *belongs to four distinct* 8*-cycles of the following canonical form:*

$$C_8 = r_n r_{n-1} r_{n-2} r_{n-1} r_n r_{n-1} r_{n-2} r_{n-1}.$$

There are $\frac{n!}{2}$ *distinct* 8*-cycles in* $P_n^{big_3}$, $n \geqslant 6$.

Theorem 3. *The* 3*-Big Pancake graph* $P_n^{big_3}$ *contains cycles of the following length:*
(i) $2n$, $2(n-1)$, *and* $2n^2 - 2n$ *for any* $n \geqslant 4$*;*
(ii) $2n^2$ *for any even* $n \geqslant 4$*;*
(iii) $\frac{n^2-1}{2}$, $\frac{n^3-4}{2}$ *and* $\frac{n^3-n^2-n+1}{2}$, *for any odd* $n \geqslant 5$
(iv) $\frac{8n}{3}$ *for any* $n \geqslant 6$*, where* 3 *divides* n.

Results of Theorem 3 are based on a greedy approach [2].

Some results on hamiltonian cycles in the 3-Big Pancake graph $P_n^{big_3}$ are also discussed.

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On cardinalities of rigid modular lattices

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A lattice is called *rigid* if any its endomorphism is a constant endomorphism (mapping all elements to a some single element) or the identity endomorphism.

It is proved in [1] that for any natural $n \geq 7$ there exists a finite rigid lattice of cardinality n and also exists a countable rigid lattice. It is easy to show that doesn't exist nontrivial (not one- and two-element) rigid distributive lattices. The question naturally arises about cardinalities of rigid modular lattices.

Theorem. *For any natural n there exist a $12 + 4n$-element and a $16 + 5n$-element rigid modular lattices.*

We establish that there exist a countable rigid modular lattice.

In the proof we uses the diagram of the 16-element rigid modular lattice from [2].

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On the energy of Cayley graphs of order a product of three primes

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Babai was the first person who computed the spectrum of Cayley graphs. In the current paper, we determine the energy of Cayley graphs of order pqr where p, q and r are prime numbers in terms of theis character table. In the continuing, we determine the graph energy and Estrada index of some well-known Cayley graphs.

On the spectrum of derangement graphs of order a product of three primes

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Let $[n] = \{1, 2, \ldots, n\}$. A subset S of a permutation group G is said to be intersecting if for any pair of permutations $\sigma, \tau \in S$ there exists $i \in [n]$ such that $\sigma\tau^{-1}(i) = i$. A derangement is a permutation with no fixed points. A subset $\mathcal{D}$ of a permutation group is derangement if all elements of $\mathcal{D}$ are derangement. Suppose G is a permutation group and $\mathcal{D} \subseteq G$ is a derangement. The derangement graph $X_{\mathcal{D}} = C(G, \mathcal{D})$ has the elements of G as its vertices and two vertices are adjacent if and only if they do not intersect. Since $\mathcal{D}$ is a union of conjugacy classes, $X_{\mathcal{D}}$ is a normal Cayley graph. In this paper, we compute the spectrum of some well-known derangement graphs.

Intervals in subgroup lattices of locally finite groups

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The author with J. Tuma proved in [1] that every algebraic lattice with at most countably many compact elements is isomorphic to an interval in the subgroup lattice of a countable locally finite group. We proved that the statement remains true if even the "countability" in it is replaced by an arbitrary infinite cardinality. In the corresponding proof we essentially use both group theoretical techniques and methods of the paper [2].

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About Aleshin's group

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In the article [2] Aleshin's construction [1] is extended to all prime numbers. In this article, the sufficient condition of periodicity of the constructed groups is proved.

In 1986, R. I. Grigorchuk raised a question of finding of necessary conditions of periodicity. In this work this question is resolved.

Let p be a prime, V be the vector space $GF(p)^p$, $\Xi = \{0,1\}^p$.

Definition. *Let* $v = (v_1, v_2, ..., v_p) \in V$, $\xi = (\xi_1, \xi_2, ..., \xi_p) \in \Xi$. *A subset* $\Sigma \subseteq \Xi$ *is called* **canceling** *if for any* $v \in V$ *there exists* $\xi \in \Sigma$ *such that* $(\xi, v) = \xi_1 v_1 + \xi_2 v_2 + ... + \xi_p v_p = 0$. *A canceling set is called* **minimal** *if its any proper subset is not canceling.*

The sufficient condition of periodicity [2] claims that the sequence of sets setting one of generating contains infinitely many elements of some **canceling** set. The necessary condition is a complete description of all minimal **canceling** sets.

Theorem 1. *Let* Σ *be the following subset of* Ξ : *it contains* p *vectors with one non-zero coordinate* $(1, 0, ..., 0)$, $(0, 1, 0, ..., 0)$, ..., $(0, ..., 0, 1)$; $p - 1$ *vectors with two non-zero coordinates* $(1, 1, 0, ..., 0), (0, 1, 1, 0, ..., 0), ..., (0, ..., 0, 1, 1)$ *etc. ... and one vector* $(1, 1, ..., 1)$. *Then* $|\Sigma|$ *is equal to* $\frac{p(p+1)}{2}$ *and* Σ *is minimal canceling set.*

Theorem 2. *Any minimal canceling subset of* Ξ *has the cardinality* $\frac{p(p+1)}{2}$. *Also it turns out from initial the* **canceling** *set application of some permutation from* S_n.

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On abelian Schur groups of odd order

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Joint work with Ilya Ponomarenko

Let G be a finite group. If Γ is a permutation group with $G_{right} \leq \Gamma \leq Sym(G)$ and $\mathcal{S}$ is the set of orbits of the stabilizer of the identity $e = e_G$ in Γ, then the $\mathbb{Z}$-submodule $\mathcal{A}(\Gamma, G) = Span_{\mathbb{Z}}\{\underline{X} : X \in \mathcal{S}\}$ of the group ring $\mathbb{Z}G$ is an S-ring as it was observed by Schur. Following Pöschel an S-ring $\mathcal{A}$ over G is said to be *schurian* if there exists a suitable permutation group Γ such that $\mathcal{A} = \mathcal{A}(\Gamma, G)$. A finite group G is called a *Schur group* if every S-ring over G is schurian.

From the results proved in [1, 2] it follows that all abelian Schur groups of odd order are known except for the groups $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_p$, where p is an odd prime. We prove the following

Theorem 1. For every prime p the group $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_p$ is Schur.

All cyclic Schur groups were classified in [3]. Theorem 1 and previously obtained results yield the complete classification of all abelian noncylic Schur groups of odd order.

Theorem 2. A noncyclic abelian group of odd order is Schur if and only if it is isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_{3^k}$ or $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_p$, where p is an odd prime.

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Using Sat solvers for synchronization issues in nondeterministic automaton

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We consider synchronization of nondeterministic finite automata (NFAs). For NFAs, there are three versions of synchronization in the literature, see [1]. Let $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ be an NFA, with Q being the states set, Σ being the input alphabet, and $\delta \subseteq Q \times \Sigma \times Q$ being the transition relation. For a state $q \in Q$ and a word w over Σ, we write $q.w$ for the set $\{p \mid (q, w, p) \in \delta\}$ and let $Q.w = \bigcup_{q \in Q} q.w$. The automaton $\mathscr{A}$ is said to be D_i-*synchronizing* $(i = 1, 2, 3)$ if there is a word w that satisfies one of the following conditions:

D_1**-synchronization:** $\forall q \in Q\ q.w \neq \varnothing$ and $|q.w| = |Q.w| = 1$;

D_2**-synchronization:** $\forall q \in Q\ q.w = Q.w \neq \varnothing$;

D_3**-synchronization:** $\bigcap_{q \in Q} q.w \neq \varnothing$.

All these notions concide with the usual notion of synchronization for deterministic automata, see [3], but they are different for NFAs.

In this talk we present an approach to studying D_3-synchronization of NFAs that uses an encoding of this property as a SAT-formula and invoking a SAT-solver. A similar approach for the deterministic case was explored by Skvortsov and Tipikin [2]; however, the case of NFAs is much more involved. Given an NFA $\mathscr{A}$ and a positive integer ℓ, our algorithm determines whether or not $\mathscr{A}$ has a D_3-synchronizing word of length ℓ.

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Eigenfunctions of the largest non-principal eigenvalue $n-2$ in the Star graphs

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Joint work with Sergey Goryainov, Vladislav Kabanov, Elena Konstantinova and Alexander Valyuzhenich

We investigate eigenfunctions of the largest non-principal eigenvalue $n-2$ in the Star graphs S_n, $n \geqslant 2$, defined as Cayley graphs on the symmetric group $\mathrm{Sym_n}$ generated by the set of transpositions $\{(1\ 2), (1\ 3), \ldots, (1\ n)\}$. The graph S_n, $n \geqslant 2$, has integral spectrum containing all integers from the set $\{-(n-1), \ldots, n-1\}$ (with the sole exception when $n \leqslant 3$, zero is not an eigenvalue of S_n) [2,3]. Moreover, the spectrum is symmetric with multiplicities $\mathrm{mul}(n-k) = \mathrm{mul}(-n+k)$ for each integer $1 \leqslant k \leqslant n$, and $\pm(n-1)$ is a simple eigenvalue. In [1] it is proved that $mul(n-2) = (n-1)(n-2)$ for any $n \geqslant 3$.

We define the set $\mathcal{F}_2 = \{f_i^{2,k} \mid i \in \{2, 3, \ldots, n\}, k \in \{3, 4, \ldots, n\}\}$, where

$$f_i^{j,k}(\pi) = \begin{cases} 1, & \text{if } \pi_j = i; \\ -1, & \text{if } \pi_k = i; \\ 0, & \text{otherwise,} \end{cases}$$

for any $\pi = [\pi_1 \pi_2 \ldots \pi_n] \in \mathrm{Sym_n}$.

We define a matrix M_n such that:

- rows are indexed by a sequence of eigenfunctions from $\mathcal{F}_2$;
- columns are indexed by a sequence of permutations from the second neighbourhood of the identity permutation $N_2 = \{(1rs) \mid r, s \in \{2, \ldots, n\}, r \neq s\}$, $|N_2| = (n-1)(n-2)$;
- the entries are values of row eigenfunctions on the corresponding column permutations.

Theorem. $|det(M_n)| = (n-2)^{n-2}(n^2 - 5n + 5)$.

The theorem immediately gives us the following result.

Corollary. *The set $\mathcal{F}_2$ forms a basis of the eigenspace of S_n, $n \geqslant 4$, corresponding to the largest non-principal eigenvalue $n-2$.*

A generalization of this approach to constructing eigenfunctions of S_n corresponding to the eigenvalue $n-m-1$, for any positive integers m and n, $2m < n$, $n \geqslant 4$, is discussed in the talk.

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On groups saturated by dihedral groups and linear groups of degree two

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A group H is saturated by groups from a set X of groups if any finite subgroup K of H is isomorphic to a subgroup of some $G \in X$. Groups with saturation conditions were studied by different authors (see, for example, a review [1]).

Let $\mathfrak{A} = \{L_2(q),\ q > 3,\ q = \pm 3 \pmod 8\}$, $\mathfrak{B}$ be the set of finite dihedral groups with Sylow 2-subgroups of order 2, and $\mathfrak{M} = \mathfrak{A} \cup \mathfrak{B}$.

The following results were obtained.

Theorem 1. *A periodic group G saturated by groups from the set $\mathfrak{M}$ is isomorphic to either a group $L_2(Q)$ for a suitable locally finite field Q or a locally dihedral group with Sylow 2-subgroups of order 2.*

Theorem 2. *The Shunkov group G saturated by groups from the set $\mathfrak{M}$ has a periodic part $T(G)$ that is either isomorphic to a group $L_2(Q)$ for a suitable locally finite field Q or a locally dihedral group with Sylow 2-subgroups of order 2.*

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On lattices with left modular and distributive elements among generators

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An element a of a lattice L is called *left modular* if

$$\forall x, b \in L : x < b \to x \vee (a \wedge b) = (x \vee a) \wedge b.$$

The study of the left modular elements is inspired by the following fact: the normal subgroups (ideals) are left modular elements in the subgroup (subring) lattice.

This property is a modular analogue of the standard element of a lattice ([1]). It is easy to verify every standard element is left modular. Moreover, the element is standard if and only if it is left modular and distributive. So the element is neutral if and only if it is left modular, distributive, and dually distributive.

The 3-generated lattice with one neutral generator is distributive and therefore this lattice is finite.

We proved the following theorems.

Theorem 1. *Let L be a 3-generated lattice in which one generator is left modular and another generator is distributive and dually distributive. Then L is finite and contains not more than 29 elements.*

In the same time the following theorem is true.

Theorem 2. *There exists an infinite 3-generated lattice in which one generator is left modular and two other generators are distributive.*

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Some examples of three-dimensional maps

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The author of the paper [1] defines the notion of an n-dimensional map, related notions and properties as cells, boundaries, duality and others applying them to Geometric Modeling. In this paper, we have a different approach to the interpretation of the three-dimensional map notion and it allows us to clarify the connection with covering mapping and apply it further on in map counting. Moreover, we provide examples of three-dimensional maps as well as some observations and remarks.

Let Σ be the cellular decomposition of a three-dimensional closed orientable manifold M, and Σ' is its barycentric subdivision. Since M is orientable there exists a checkerboard colouring of tetrahedra from Σ'. We will use black and white colours to make such a colouring. Denote by $F^{\pm}$ and F the set of all colored tetrahedra and the set of only black tetrahedra, respectively. Let A, B, C, D be vertices of one of tetrahedra from F as well as denoting in the same way the reflections in its corresponding faces. Then elements $a = AC, b = AD, c = BD$ are the order two rotations in its corresponding faces sending the black (white) tetrahedra to the black (white) ones. Let $\Delta =< A, B, C, D >$ be the group generated by reflections A, B, C, D. Let us denote by Δ^+ the subgroup of the index two in Δ generated by the elements a, b, c. The group Δ acts on $F^{\pm}$ while Δ^+ acts on F. So we get two homomorphisms φ: $\Delta \to Sym(F^{\pm})$ and φ^+: $\Delta^+ \to Sym(F)$.

Definition: *Three-dimensional map (or 3-map) is a four-tuple $(F; a, b, c)$ with the following properties:*
1° *F is the finite set of flags;*
2° *a, b, c are involutions acting without fixed points freely on F;*
3° *The group Δ^+ is transitive on the set F;*
4° *Orbits of the permutation ab are edges, bc — faces, $\langle a, bc\rangle$ — cells and $\langle ab, c\rangle$ — vertices of the 3-map.*

This definition can be illustrated by the following examples: «pillow» with 4 marked vertices, «shell» with 2 marked vertices, a cone with 3 marked vertices, a quadrilateral face with 4 marked vertices, and others.

Remark. *The Reidemeister-Schreier method one can show that the group $\Delta^+ = \langle a, b, c : a^2 = b^2 = c^2 = 1\rangle$ is freely generated by three involution a, b and c. Hence, we can replace a, b, c by their images $\varphi^+(a)$, $\varphi^+(b)$, $\varphi^+(c)$ in the definition of the 3-map.*

The following statement is correct:

Proposition. *There exists a one-to-one correspondence between the orbits on $F^{\pm}$ and three-dimensional map elements: the orbits $\langle A, B\rangle$ correspond to the faces, $\langle C, D\rangle$ to the edges, $\langle A, B, C\rangle$ to the cells, and $\langle B, C, D\rangle$ to the vertices of the 3 - map.*

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Structure of commutator subgroups and centralizers of Sylow 2-subgroups of alternating and symmetric groups, minimal generating sets of Sylow 2-subgroups of alternating groups, and applications to cryptography

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In this talk we discuss a structure of commutator subgroups and centralizers of Sylow 2-subgroups of alternating and symmetric groups, minimal generating sets of Sylow 2-subgroups of alternating groups, and applications to cryptography.

The $\mathfrak{F}_\Omega$-covering subgroups and $\mathfrak{F}_\Omega$-projectors of finite groups

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We consider finite groups only. We have introduced definitions of an $\mathfrak{F}_\Omega$-covering subgroup and of an $\mathfrak{F}_\Omega$-projector in a finite group G which are the generalization of Gaschütz's definitions of an $\mathfrak{F}$-covering subgroup and of an $\mathfrak{F}$-projector, respectively.

Let $\mathfrak{I}$ be the class of all simple groups, Ω be a non-empty subclass of $\mathfrak{I}$. A group G is called an Ω-group if $K(G) \subseteq \Omega$, where $K(G)$ is the set of all composition factors of G. Let $\mathfrak{F}_\Omega$ be the class of all Ω-groups belonging the class $\mathfrak{F}$.

Definition 1. Let $\mathfrak{F}$ be a non-empty class of groups. A subgroup H of the group G is called an $\mathfrak{F}_\Omega$-covering subgroup of G if $H \in \mathfrak{F}_\Omega$ and whenever $H \le U \le G$, and V is a normal Ω-subgroup of U such that $U/V \in \mathfrak{F}_\Omega$, then $U = HV$.

Definition 2. Let $\mathfrak{F}$ be a non-empty class of groups. A subgroup H of the group G is called an $\mathfrak{F}_\Omega$-projector of G if HN/N is an $\mathfrak{F}_\Omega$-maximal subgroup in G/N for every normal Ω-subgroup N of G.

Let $\mathfrak{F}$ and $\mathfrak{X}$ be non-empty classes of groups such that $\mathfrak{F} \subseteq \mathfrak{X}$. Following [1], we say that a class $\mathfrak{F}$ is Ω-primitively closed in $\mathfrak{X}$, or briefly, ΩP-closed in $\mathfrak{X}$ if for each group G the following condition is satisfied: if $G/Core_G(M) \cap O_\Omega(G) \in \mathfrak{F}$ for every maximal subgroup M of G, then $G \in \mathfrak{F}$.

Theorem 1. *Let $\mathfrak{X}$ be an S-closed homomorph, $\mathfrak{F}$ be a non-empty ΩP-closed homomorph in $\mathfrak{X}$ and $G \in \mathfrak{X}$. If G has a solvable $\mathfrak{F}_\Omega$-residual Ω-subgroup, then there exists at least one $\mathfrak{F}_\Omega$-projector of G.*

Theorem 2. *Let $\mathfrak{X}$ be an S-closed homomorph and $\mathfrak{F}$ be a non-empty subclass of $\mathfrak{X}$. If every group $G \in \mathfrak{X}$ with $O_\Omega(G) \ne 1$ has an $\mathfrak{F}_\Omega$-projector, then the following assertions hold*:
(1) *$\mathfrak{F}_\Omega$ is ΩP-closed in $\mathfrak{X}$*;
(2) *if $N \in \mathfrak{F}_\Omega$ and N is a normal Ω-subgroup of G, then $H/N \in \mathfrak{F}_\Omega$.*

Theorem 3. *Let $\mathfrak{X}$ be an S-closed homomorph, $\mathfrak{F}$ be a non-empty ΩP-closed homomorph in $\mathfrak{X}$, $G \in \mathfrak{X}$ and N be a nilpotent normal Ω-subgroup of G. If H is an $\mathfrak{F}_\Omega$-subgroup of G with $G = HN$ then H lies in an $\mathfrak{F}_\Omega$-projector of G. In particular, if H is an $\mathfrak{F}_\Omega$-maximal subgroup of G then H is an $\mathfrak{F}_\Omega$-projector of G.*

Theorem 4. *Let $\mathfrak{X}$ be an S-closed homomorph, $\mathfrak{F}$ be a non-empty ΩP-closed homomorph in $\mathfrak{X}$ and G be an $\mathfrak{X}$-group which has a solvable $\mathfrak{F}_\Omega$-residual Ω-subgroup. A solvable subgroup H of G is an $\mathfrak{F}_\Omega$-projector of G if and only if H is an $\mathfrak{F}_\Omega$-covering subgroup of G.*

Let $f : \Omega \cup \{\Omega'\} \to$ {formations of groups} and $\varphi : \mathfrak{I} \to$ {non-empty Fitting formations of groups} be functions. A formation $\Omega F(f, \varphi) = (G : G/O_\Omega(G) \in f(\Omega')$ and $G/G_{\varphi(A)} \in f(A)$ for all $A \in \Omega \cap K(G))$ is called an Ω-foliated formation with the Ω-satellite f and the direction φ [2]. A formation $\mathfrak{F} = \Omega F(f, \varphi)$ is called Ω-composition if $\varphi(A) = \mathfrak{S}_{cA}$ for any $A \in \mathfrak{I}$, where $\mathfrak{S}_{cA}$ is the class of all those groups in which every chief A-factor is central.

Theorem 5. *Let $\mathfrak{F}$ be an Ω-composition formation, G be a group and $G^{\mathfrak{F}_\Omega}$ be a solvable $(\Omega \cap K(\mathfrak{F}))$-group. Then any two $\mathfrak{F}_\Omega$-covering subgroups of G are conjugate.*

Remark. Theorems 1–5 continue the researches from [1].

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On the $\mathfrak{F}$-coradicals of finite groups

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A concept of an f-central chief factor of a group is used widely in the theory of classes of finite groups (see, for example, [1]). Following [1], we defined a concept of an f_Ω-central chief factor of a group. Using the properties of f_Ω-central chief factors we obtained a new characterization of an Ω-composition formation of groups and established new properties of an $\mathfrak{F}$-coradical of a group where $\mathfrak{F}$ is an Ω-composition formation.

We consider finite groups only. Our descriptions and notations could be found in [1]. Let $\mathfrak{I}$ be the class of all simple groups, Ω be a non-empty subclass of $\mathfrak{I}$. A group G is called an Ω-group if $K(G) \subseteq \Omega$, where $K(G)$ is the set of all composition factors of G. Let $\mathfrak{E}_\Omega$ be the class of all Ω-groups, $O_\Omega(G)$ be the $\mathfrak{E}_\Omega$-radical of the group G. Let $f : \Omega \cup \{\Omega'\} \to \{\text{formations of groups}\}$ and $\varphi : \mathfrak{I} \to \{\text{non-empty Fitting formations of groups}\}$ be an ΩF-function and an FR-function respectively. These functions take the same values on isomorphic groups from their domain. A formation $\Omega F(f,\varphi) = (G : G/O_\Omega(G) \in f(\Omega')$ and $G/G_{\varphi(A)} \in f(A)$ for all $A \in \Omega \cap K(G))$ is called an Ω-foliated formation with the Ω-satellite f and the direction φ [2]. A formation $\mathfrak{F} = \Omega F(f,\varphi)$ is called Ω-composition if $\varphi(A) = \mathfrak{S}_{cA}$ for any $A \in \mathfrak{I}$, where $\mathfrak{S}_{cA}$ is the class of all those groups in which every chief A-factor is central. We say that a formation $\mathfrak{F}$ is Ω-saturated whenever, given a group G and an arbitrary normal subgroup N of G with $N \le \Phi(G) \cap O_\Omega(G)$, the property $G/N \in \mathfrak{F}$ implies that $G \in \mathfrak{F}$.

Definition 1. Let f be an ΩF-function. Following [1], we say that a chief Ω-factor M/N of a group G is f_Ω-central in G if $G/C_G(M/N) \in f(A)$ for any $A \in K(M/N)$.

Theorem 1. *Let $\mathfrak{F}$ be an Ω-composition formation with an inner Ω-satellite f, let $\mathfrak{H}$ be the class of all those groups G that $G/O_\Omega(G) \in f(\Omega')$ and every chief Ω-factor of G is f_Ω-central in G. Then $\mathfrak{F} = \mathfrak{H}$.*

In the following two theorems we established sufficient conditions in which an $\mathfrak{F}$-coradical of a group G doesn't have f_Ω-central G-chief factors, where f is an inner Ω-satellite of an Ω-composition formation.

Theorem 2. *Let $\mathfrak{F}$ be an Ω-composition formation with an inner Ω-satellite f, $Z_p \in \Omega$ and let G be a group. Suppose that a Sylow p-subgroup P of $G^{\mathfrak{F}}$ is abelian and one of the following two conditions hold:*

(1) P lies in the Frattini subgroup of a solvable normal subgroup of G;

(2) $\mathfrak{F}$ is a Ω-saturated formation.

Then $G^{\mathfrak{F}}$ doesn't have f_Ω-central G-chief p-factors.

Theorem 3. *Let $\mathfrak{F}$ be an Ω-composition Fitting formation with a maximal inner Ω-satellite f and let $G = A_1A_2\cdots A_n$ be a group where $A_1, A_2, \dots, A_n$ are pairwise commuting subnormal subgroups of G. Suppose that for every simple group $Z_p \in K(\mathfrak{F}) \cap \Omega$ a Sylow p-subgroup P_i of $A_i^{\mathfrak{F}}$ is abelian for each $i \in \{1, 2, \dots, n\}$ and one of the following two conditions hold:*

(1) P_i lies in the Frattini subgroup of a solvable normal subgroup of A_i for each $i \in \{1, 2, \dots, n\}$;

(2) $\mathfrak{F}$ is a Ω-saturated formation.

Then $G^{\mathfrak{F}} = A_1^{\mathfrak{F}} A_2^{\mathfrak{F}} \dots A_n^{\mathfrak{F}}$ and $G^{\mathfrak{F}}$ doesn't have f_Ω-central G-chief p-factors.

Remark. These results continue the authors' researches from [3].

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On characterization of alternating and symmetric groups by prime graph

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Let G be a finite group. The *spectrum* $\omega(G)$ of G is the set of its element orders. The set of prime divisors of $|G|$ is denoted by $\pi(G)$. The spectrum defines the *prime graph* (or the *Gruenberg* − *Kegel graph*) $GK(G)$ of G: the set of vertices is $\pi(G)$, and two distinct vertices r and s are adjacent if and only if $rs \in \omega(G)$. Let Alt_n, Sym_n denote the alternating and symmetric groups of degree n, respectively. It was proved in [1] that if G is a finite group such that $\omega(G) = \omega(Alt_n)$ with $n \geq 5$ and $n \neq 6, 10$ then $G \simeq Alt_n$. We study finite groups G such that $GK(G) = GK(Alt_n)$ or $GK(G) = GK(Sym_n)$ and prove the following.

Theorem. *Let $n \geq 41$ be an integer and p the largest prime less than or equal to n. If G is a finite group such that $GK(G) = GK(Alt_n)$ or $GK(G) = GK(Sym_n)$, then there exists a normal subgroup K of G such that $Alt_t \leq G/K \leq Sym_t$, where $t \geq p$.*

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On automorphisms of $AT4(5,7,3)$-graphs

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A non-bipartite antipodal distance-regular graph Γ of diameter 4 is called tight whenever the equality in the fundamental bound is attained. If Γ is tight, then its intersection array is expressed in terms of the non-trivial eigenvalues p and $-q$ of the local graphs and the size r of its antipodal class, and Γ is denoted by $AT4(p,q,r)$. Many known examples of $AT4(p,q,r)$-graphs have small value of $|p-q|$.

In 2016, A. Makhnev and D. Paduchikh restricted admissible parameter sets of $AT4(p,p+2,r)$-graphs of valency at most 1000 [1, Theorem 3]. The second Soicher graph with intersection array $\{56,45,16,1;1,8,45,56\}$ provides the only known example of graphs with these parameters. Besides, it is the unique $AT(2,4,3)$-graph.

In this work, we study automorphisms of a hypothetical $AT4(5,7,3)$-graph. This graph is a distance-regular 3-cover of a strongly regular graph with parameters $(1458,329,40,84)$ and its second subconstituent is a distance-regular graph with intersection array $\{245,216,40,1;1,20,216,245\}$. In particular, we analyse automorphisms of the antipodal quotient and the second subconstituent of an $AT4(5,7,3)$-graph.

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Minimal supports of eigenfunctions of Hamming graphs

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Many combinatorial configurations (for example, perfect codes, latin squares and hypercubes, combinatorial designs and their q-ary generalizations — subspace designs) can be defined as an eigenfunction on a graph with some discrete restrictions. The study of these configurations often leads to the question about the minimum possible difference between two configurations from the same class (it is often related with bounds of the number of different configurations; for example, see [1–5]). Since the difference of characteristic functions of these two configurations is also an eigenfunction, this question is directly related to the minimum cardinality of the support (the set of nonzero) of an eigenfunction with given eigenvalue. This paper is devoted to the problem of finding the minimum cardinality of the support of eigenfunctions in the Hamming graphs $H(n,q)$. It is well-known that the set of eigenvalues of the adjacency matrix of $H(n,q)$ is $\{\lambda_m = n(q-1) - qm \mid m = 0,1,\ldots,n\}$. Currently, this problem is solved only for $q = 2$ (see [4]). In [7] Vorob'ev and Krotov proved the lower bound on the cardinality of the support of an eigenfunction of the Hamming graph. In [6] this problem was solved for the second largest eigenvalue $n(q-1) - q$. In this paper we solve this problem for all eigenvalues of the Hamming graphs $H(n,q)$ and $q \geq 4$.

We prove the following theorem:

Theorem 1. *Let $f : H(n,q) \longrightarrow \mathbb{R}$ be an eigenfunction corresponding to λ_i, $q \geq 4$ and $f \not\equiv 0$.*

1. *Then $|S(f)| \geq 2^i(q-1)^i q^{n-2i}$ for $i \leq \lfloor n/2 \rfloor$.*
2. *Then $|S(f)| \geq 2^i(q-1)^{n-i}$ for $i > \lfloor n/2 \rfloor$.*

Moreover, we describe the set of functions with the minimum cardinality of the support for the first case of theorem for $q \geq 4$ and for the second case for $q > 4$.

For the first case of Theorem 1 we obtain more general result. Denote by $U_j(n,q)$ an eigenspace corresponding to λ_j. The space $U_i(n,q) + \ldots + U_j(n,q)$ for $i \leq j$ is denoted by $U_{[i,j]}(n,q)$. We prove the following theorem:

Theorem 2. *Let $f : H(n,q) \longrightarrow \mathbb{R}$, $f \in U_{[i,j]}(n,q)$ and $f \not\equiv 0$. Then $|S(f)| \geq 2^i(q-1)^i q^{n-i-j}$ for $n \geq i+j$ and $q \geq 4$.*

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On commutator subgroups of finite 2-groups generated by involutions

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We denote the commutator subgroup of any group G by G' and the minimal number of generators of G by $d(G)$.

Ustyuzhaninov [1] without proof had presented the list of finite 2-groups generated by 3 involutions with elementary abelian commutator subgroup. In particular, $d(G') \leq 5$ for any such group G.

As a continuation of this theme it is interesting to obtain the classification of finite 2-groups generated by n involutions (for any $n \geq 2$) with elementary abelian commutator subgroup.

We prove the following theorem.

Theorem. If a finite 2-group G is generated by n involutions then for any $n \geq 2$

$$d(G') \leq \binom{n}{2} + 2\binom{n}{3} + \cdots + (n-1)\binom{n}{n},$$

and this inequality may not be improved.

For any $n \geq 2$ we construct a finite 2-group G generated by n involutions with elementary abelian commutator subgroup of rank equal to $\binom{n}{2} + 2\binom{n}{3} + \cdots + (n-1)\binom{n}{n}$ by means of Lemma 1 from [2].

In addition, on the base of constructed 2-groups we obtain an example of non-nilpotent infinite 2-group generated by involutions with infinite elementary abelian commutator subgroup.

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Cancellable elements of the lattice of semigroup varieties

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This is the joint work with S.V.Gusev and D.V.Skokov

There are a number of articles devoted to an examination of special elements of different types in the lattice **SEM** of all semigroup varieties (see the recent survey [4]). This work continues these studies. An element $x \in L$ is called

modular if $(\forall y, z \in L) \quad y \leq z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y;$

cancellable if $(\forall y, z \in L) \quad x \vee y = x \vee z \ \&\ x \wedge y = x \wedge z \longrightarrow y = z.$

It is easy to see that any cancellable element is a modular one. A valuable information about modular elements of **SEM** were obtained in [1–3]. In particular, commutative semigroup varieties that are modular elements of **SEM** were completely determined in [3, Theorem 3.1]. Cancellable elements of **SEM** were not examined so far.

We prove the following

Theorem. *For a commutative semigroup variety* $\mathbf{V}$*, the following are equivalent*:

a) $\mathbf{V}$ *is a cancellable element of the lattice* **SEM**;

b) $\mathbf{V}$ *is a modular element of the lattice* **SEM**;

c) $\mathbf{V} = \mathbf{M} \vee \mathbf{N}$ *where* $\mathbf{M}$ *is either the trivial variety or the variety of all semilattices, while* $\mathbf{N}$ *is a variety satisfying the identities* $x^2y = 0$ *and* $xy = yx$.

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On the volume of hyperbolic tetrahedron with symmetry group S_4

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We consider a hyperbolic tetrahedron with symmetry group C_2 by Schönflies notation (or 2-symmetry by Hermann-Mauguin notation). By definition, a tetrahedron has 2-symmetry if it admits a π rotation around the axis passing through the middles of two opposite edges. In particular case when $a = b$, $\theta = \pi/2$ (and $c = d$) we get a tetrahedron with symmetry group S_4 (or $\overline{4}$-symmetry). We establish a criterion of existence for such a tetrahedron and obtain exact formula for its hyperbolic volume.

Theorem 1. *A hyperbolic tetrahedron with edge lengths a, c admitting $\overline{4}$-symmetry is exist if and only if* $1 + cha - 2chc < 0$.

Theorem 2. *The volume of a hyperbolic tetrahedron with edge lengths a, c admitting $\overline{4}$-symmetry is given by the formula*

$$V = \int_0^a f(a,c)\, da = \int_{arch((1+cha)/2)}^{c} g(a,c)\, dc, \ where$$

$$f(a,c) = -a\frac{1}{\sqrt{A_2^2 - A_1^2}}\frac{A_3}{A_2}sha - 2c\frac{1}{\sqrt{A_2^2 - C_1^2}}\frac{C_2}{A_2}sha,$$

$$g(a,c) = -a\frac{1}{\sqrt{A_2^2 - A_1^2}}\frac{A_4}{A_2}shc - 2c\frac{1}{\sqrt{A_2^2 - C_1^2}}\frac{C_3}{A_2}shc,$$

$$A_1 = (1 - cha)(1 + cha)^3 - 16(1 + cha)^2 ch^2c + 64ch^4c,$$

$$A_2 = (cha - 1)(1 + cha)^3 - 16(1 + cha)^2 ch^2c + 64ch^4c,$$

$$A_3 = 64ch^2c\,(1 + cha)^2[cha(1 + cha)^2 + 4(1 - 2cha)ch^2c],$$

$$A_4 = 64chc\,(1 - cha)(1 + cha)^3[(1 + cha)^2 - 8ch^2c],$$

$$C_1 = (ch^2a - 1)[(1 + cha)^2 - 8ch^2c],$$

$$C_2 = 16ch^2c\,(1 + cha)^3[1 - cha(4 + cha)]+$$
$$8(1 + cha)^2(1 + 2cha)^2(1 + 2cha)^2(1 + 2cha)ch^2c - 64chach^4c,$$

$$C_3 = 16chc[(1 - cha)(1 + cha)^3 + 16(1 + cha)^2ch^2c - 64ch^4c]+$$
$$2(1 - ch^2a)(1 + cha)(cha - 1 + 2ch^2a - 16ch^2c)[(1 + cha)^2 - 8ch^2c].$$

We also obtain existence criteria and volume formulas in more general cases when $a \neq b$ or $\theta \neq \pi/2$ ($c \neq d$).

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On intersection of two nilpotent subgroups in a finite group with the socle isomorphic to $G_2(3)$, $G_2(4)$, $F_4(2)$, ${}^2F_4(2)'$, $Sz(8)$, $E_6(2)$, or ${}^2E_6(2)$

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In [1, theorem B(2b)] it was proved that if $K \in \{G_2(3), G_2(4), F_4(2), {}^2F_4(2)', Sz(8), E_6(2), {}^2E_6(2)\}$, $G = Aut(K)$, and P and Q are Sylow subgroups of K then there exists an element $g \in G$ such that $P \bigcap Q^g = 1$, except of the case when $P, Q \in Syl_2(G)$ or $P, Q \in Syl_3(G)$.

We prove the following theorem.

Theorem. *Let $K \in \{G_2(3), G_2(4), F_4(2), {}^2F_4(2)', Sz(8), E_6(2), {}^2E_6(2)\}$ and $G = Aut(K)$. If A and B are nilpotent subgroups of G and $A \bigcap B^g \neq 1$ for every $g \in G$ then $K \in \{F_4(2), E_6(2)\}$, and A and B are 2-subgroups.*

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On finite simple linear groups over fields of different characteristics with coinciding prime graphs

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For a positive integer n, denote by $\pi(n)$ the set of prime divisors of n. Given a finite group G, write $\pi(G)$ for $\pi(|G|)$ and $\omega(G)$ for the set of element orders of G. The prime graph $GK(G)$ of G is an ordinary graph with the vertex set $\pi(G)$ in which two distinct vertices r and s are adjacent if and only if $rs \in \omega(G)$.

In [1], A. V. Vasil'ev posed the question 16.26. We can consider this question as the question about a description of all pairs of non-isomorphic finite simple groups with the same prime graph. M. Hagie [2] and M. A. Zvezdina [3] got such a description in case one of groups is sporadic and alternating, respectively.

In [4], the author obtained the description of pairs of non-isomorphic groups of Lie type over fields of the same characteristics with coinciding prime graphs. In [5], it is proved that, if G is a classical group of a sufficiently high Lie rank, G_1 is a finite simple group of Lie type, non-isomorphic to G, and G and G_1 have different characteristics, then the prime graphs of the groups G and G_1 may coincide only in one of three cases.

In this work we continue the investigation of finite simple classical groups over fields of different characteristics with coinciding prime graphs and specify the description from [5] in case one of groups is a linear group $L_n(q)$, where $n \geq 7$. As a corollary of this description, it may be shown that for a given simple linear group $G = L_n(q)$, where $n \geq 7$, the number of finite simple groups G_1 with $GK(G_1) = GK(G)$ is finite.

This work was supported by the Russian Science Foundation (project no.15-11-10025).

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Graphs and Groups, Representations and Relations

Novosibirsk, Russia, August, 6–19, 2018

Announcement

Sobolev Institute of Mathematics of Siberian Branch of Russian Academy of Sciences and Novosibirsk State University organize the International Conference and PhD-Master Summer School "Graphs and Groups, Representations and Relations"(G2R2). It will be held in Akademgorodok, Novosibirsk, Russia, August, 6–19, 2018.

The main goal of this international interdisciplinary event is to bring together researchers from different fields of mathematics and its applications mainly based on graph theory and group theory, especially those involving group actions on combinatorial objects.

The scientific program of G2R2 includes:

- Lectures of main speakers
- Short contributions in sections
- Minicourses in the frame of the PhD-Master Summer School

The official language of G2R2 is English.

Summer School Minicourses will be given by:

Gareth Jones
University of Southampton, UK

Akihiro Munemasa
Tohoku University, Japan

Mikhail Muzychuk
Netanya Academic College, Israel

Roman Nedela
University of West Bohemia, Czech Republic

The International Conference and PhD Summer School «Groups and Graphs, Metrics and Manifolds» 2017

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